Math 1050
Test 3 (Practice)
Solutions

1. The sequence $\{a_n\}$ is defined recursively by $a_1 = -4$, $a_2 = 3$, and $a_n = -5a_{n-1} + 3a_{n-2}$. Find a_3 and a_8 .

$a_3 = -5a_2 + 3a_1 = -27$
$a_8 = 136,251$

2. Express the following sum in sigma notation:

8 + 11 + 14 + 17 + … + 176

$\sum_{n=1}^{57}(3n+5)$

3. a) Find the common difference and the 75-th term for the following arithmetic sequence.

27 , 23 , 19 , 15 , …

$d = -4$
$a_{75} = a_1 + (74)d = 27 + (74)(-4) = -269$

b) Find the common ratio, the 38-th term, and the sum of the following geometric sequence.

16 , -8 , 4 , -2 , …

$r = -\frac{1}{2}$
$a_{38} = a_1 \cdot r^{37} = 16 \cdot \left(-\frac{1}{2}\right)^{37} = -1.164 \times 10^{-10}$

4. Find the first four terms in the expansion of $\left(x - \frac{3}{x}\right)^7$

$$\left(x-\frac{3}{x}\right)^7 = x^7 + \binom{7}{1}x^6\left(-\frac{3}{x}\right)^1 + \binom{7}{2}x^5\left(-\frac{3}{x}\right)^2 + \binom{7}{3}x^4\left(-\frac{3}{x}\right)^3 + \binom{7}{4}x^3\left(-\frac{3}{x}\right)^4 + \binom{7}{5}x^2\left(-\frac{3}{x}\right)^5 + \binom{7}{6}x^1\left(-\frac{3}{x}\right)^6$$

First four terms: $x^7 - 21x^5 + 189x^3 - 945x^1$

5. Find the coefficient of the x^5 term in the binomial expansion of $(5-x)^9$.

$$\binom{9}{5}5^4(-x)^5 = -78750x^5$$

6. Reduce the following system to echelon form, and find the solution set.

$$\begin{aligned} x-2y+z &= -5 \\ -x+3y-2z &= 7 \\ 3x+4y-2z &= 25 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 1 & -5 \\ -1 & 3 & -2 & 7 \\ 3 & 4 & -2 & 25 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 5 & 20 \end{bmatrix}$$

$x=3$, $y=6$, $z=4$

7. Reduce the following system to echelon form, and find the solution set.

$$\begin{aligned} x-4y+z &= 2 \\ y+2z &= 5 \\ -x+6y+3z &= 8 \end{aligned}$$

$$\begin{bmatrix} 1 & -4 & 1 & 2 \\ 0 & 1 & 2 & 5 \\ -1 & 6 & 3 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 1 & 2 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$z=k$, $y=5-2k$, $x=22-9k$

8. Find an equation for the parabola in the form $y = x^2 + bx + c$ that contains the points (2, -7) and (-3, 33) .

Substitute both point into the equation and solve the system

$2b + c = -11$
$-3b + c = 24$

$b = -7$, $c = 3$

9. Find all points of intersection of the line $x + y = 3$ with the parabola $y = x^2 + 5x + 1$.

Solve: $3 - x = x^2 + 5x + 1$

$x = -3 \pm \sqrt{11}$

10. Graph the solution to following system of inequalities:

$y \le x^2 - 4$
$x + y - 2 \le 0$