

Math 1050
Test 2 (Practice)
Solutions

1. Find all solutions (accurate to 2-decimal places) of $x^5 - 5x^3 + 6x = 0$.

$$x^5 - 5x^3 + 6x = x(x^4 - 5x^2 + 6) = x(x-3)(x-2) = 0$$

when $x = 0$, or $x = 3$, or $x = 2$.

2. Find the coefficients for the polynomial function of lowest degree, having integer coefficients (leading coefficient 1) with the zeros 2, -3, and i .

$$(x-2)(x+3)(x-i)(x+i) = x^4 + x^3 - 5x^2 + x - 6$$

3. From the graph of the function $g(x) = \frac{3x^2 - 12}{x^2 - 2x - 3}$,

(a) Find all vertical asymptotes.

$$g(x) = \frac{3x^2 - 12}{x^2 - 2x - 3} = \frac{3(x+2)(x-2)}{(x-3)(x+1)}$$

The asymptotes are $x = 3$, and $x = -1$.

(b) Find any horizontal asymptotes.

The horizontal asymptote is $y = 3$.

(c) Locate all x -intercepts.

The x -intercepts are $x = -2$ and $x = 2$.

4. Each graph below is the graph of one of the listed functions. For each graph, identify the polynomial function.

a) $f(x) = x^2(x-1)(x+2)$

b) $f(x) = x(x+1)(x-2)$

c) $f(x) = x(x+1)^2(x-2)$

d) $f(x) = x^2(x-1)^2(x-2)$

e) $f(x) = x^2(x-1)^2(x+2)$

f) $f(x) = x(x+1)^2(x+2)$



e)



a)



b)



c)

5. Given that the number $\sqrt{3}$ is one zero of the polynomial function

$$f(x) = x^4 - x^3 - 9x^2 + 3x + 18, \text{ find all zeros of } f.$$

$$(x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3 \text{ must be a factor of the polynomial.}$$

Divide $x^2 - 3$ into $f(x)$ to obtain $x^2 - x - 6$ as another factor.

The zeros are $\sqrt{3}$, $-\sqrt{3}$, 3 , and -2 .

6. A rectangular box with a top is to have a base whose length is 4 times its width and whose volume is 2400 cubic inches. Let the variable x denote the width of the box, and let $S(x)$ be the amount of material used to construct the box.

a) Express $S(x)$ as a function of x .

$$\text{Volume} = 2400 = 4x^2h$$

$$h = \frac{600}{x^2}$$

$$S(x) = 8x^2 + \frac{6000}{x}$$

b) Graph $y = S(x)$ and trace to estimate (1 decimal place accuracy) the minimum value for $S(x)$.

Minimum value of 1248 when x is approximately 7.2

7. Find the solution of $\ln(x^2 - 8x + 16) = 0$.

$$x^2 - 8x + 16 = e^0 = 1$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$x = 3 \text{ or } x = 5$$

8. Write an equation for the inverse of the function given by

$$f(x) = 4e^{2x-3} + 5 .$$

Set $x = 4e^{2y-3} + 5$ and solve for y .

$$\frac{x-5}{4} = e^{2y-3}$$

$$\ln\left(\frac{x-5}{4}\right) = 2y-3$$

$$y = f^{-1}(x) = \frac{3}{2} + \frac{1}{2}\ln\left(\frac{x-5}{4}\right)$$

9. A sample culture contains approximately 800 bacteria when first measured and 5 hours later the number has doubled to 1600.

a) Find a formula for $A(t)$, the number of bacteria at time t hours after the initial measurement.

$$A(t) = A_0 e^{kt}, \quad A_0 = 800$$

$$A(5) = 800e^{5k} = 1600$$

$$\text{Solve for } k. \quad k = \frac{1}{5} \ln \left(\frac{1600}{800} \right) = .1386$$

$$A(t) = 800e^{.1386t}$$

b) What is the number of bacteria at the end of 24 hours?

$$A(24) = 22,286$$

c) How long does it take for the number to increase to 25,000 bacteria?

$$A(t) = 800e^{.1386t} = 25000$$

$$\text{Solve for } t. \quad t = 24.8 \text{ years}$$

10. The line $x=1$ is a vertical asymptote for which of the following rational functions?

(i) $\frac{x+4}{x^2+x}$

$$\frac{x+4}{x^2+x} = \frac{x+4}{x(x+1)}, \text{ no}$$

(ii) $\frac{x^2-5x+4}{x^2-x}$

$$\frac{x^2-5x+4}{x^2-x} = \frac{(x-4)(x-1)}{x(x-1)}, \text{ no}$$

$$(iii) \quad \frac{x+1}{x^2-x}$$

$$\frac{x+1}{x^2-x} = \frac{x+1}{x(x-1)}, \text{ yes}$$

$$(iv) \quad \frac{x^2-5x+4}{x^2-2x+1}$$

$$\frac{x^2-5x+4}{x^2-2x+1} = \frac{(x-4)(x-1)}{(x-1)(x-1)}, \text{ yes}$$