

**Math 1050**  
**Test 1**  
**Solutions**

1. Give all linear factors of  $3x^3 + 2x^2 - x$ .

$$3x^3 + 2x^2 - x = x(3x^2 + 2x - 1) = x(3x - 1)(x + 1)$$

2. Rationalize the denominator of  $\frac{3}{\sqrt{2}-5}$  and simplify.

$$\frac{3}{\sqrt{2}-5} \cdot \frac{\sqrt{2}+5}{\sqrt{2}+5} = \frac{3\sqrt{2}+15}{2-25} = \frac{-3\sqrt{2}-15}{23}$$

3. Write a formula for  $f^{-1}(x)$ , the inverse of the function given by

$$f(x) = \frac{1-2x}{x+3}.$$

$$\text{Set } x = \frac{1-2y}{y+3} \text{ and solve for } y.$$

$$xy + 3x = 1 - 2y$$

$$xy + 2y = 1 - 3x$$

$$y(x+2) = 1 - 3x$$

$$y = f^{-1}(x) = \frac{1-3x}{x+2}$$

4. Let  $f(x) = 2 - x^2$ . Simplify the expression  $\frac{f(x) - f(1)}{x - 1}$ .

$$\frac{f(x) - f(1)}{x - 1} = \frac{2 - x^2 - 1}{x - 1} = \frac{1 - x^2}{x - 1} = \frac{(1 + x)(1 - x)}{x - 1} = -1 - x$$

5. Find the center and radius for the circle with equation

$$x^2 + y^2 + 8x - 2y + 1 = 0$$

$$x^2 + y^2 + 8x - 2y = -1$$

$$x^2 + 8x + y^2 - 2y = -1$$

$$x^2 + 8x + 16 + y^2 - 2y + 1 = -1 + 16 + 1$$

$$(x + 4)^2 + (y - 1)^2 = 16$$

The circle has center at  $(-4, 1)$  and radius 4.

6. Use a graphing calculator (or another graphing utility) to determine the intervals for which the

function  $f(x) = 4 - 2x - x^2$  is increasing.

The function is increasing for  $x < -1$ .

7. Write an equation in the form  $y = mx + b$  for the line through the point  $(4, 3)$  that is perpendicular

to the line  $2x - 5y = 6$ .

The equation  $2x - 5y = 6$  is equivalent to  $y = \frac{2}{5}x - \frac{6}{5}$

and so the slope of the line given by  $2x - 5y = 6$  is  $\frac{2}{5}$ .

Using the negative reciprocal property, the slope of the desired line is  $-\frac{5}{2}$  and its equation is  $y - 3 = -\frac{5}{2}(x - 4)$ .

8. Find the domain of the function  $f(x) = \sqrt{x^3 - x}$ .

We must solve the inequality  $x^3 - x \geq 0$ .

$$x(x^2 - 1) \geq 0$$

$$x(x+1)(x-1) \geq 0$$

Draw a line and label the points 0, 1, -1. The inequality is satisfied when  $-1 \leq x \leq 0$  or  $1 \leq x$ .

9. An open-topped box is to be made from a square piece of cardboard 14 inches on a side, by cutting an  $x$ " by  $x$ " square from each corner and bending up the sides. Express the volume of the resulting box in terms of  $x$ . Use a graphing calculator and the trace feature to determine the value of  $x$  for which the volume is a maximum.

$$V(x) = (14 - 2x)(14 - 2x)x$$

$V(x)$  is a maximum when  $x \approx 2.3$

10. A group of  $x$  friends want to purchase an airplane that costs 108 thousand dollars. If they can get 3 more people to join their group, the cost for each person will decrease by 6 thousand dollars. Find  $x$ .

If  $x$  friends purchase the airplane, the cost (in \$1000 s)

is  $\frac{108}{x}$ . With three more people, the cost is  $\frac{108}{x-3}$ . So,

$$\frac{108}{x+3} = \frac{108}{x} - 6. \text{ To solve, multiply both sides by } x(x+3).$$

The solution is  $x = 6$ .