Utah State University
ECE 6010
Stochastic Processes
Homework
Due Friday November 8, 2003

1. Suppose $\{X_t, t \in \mathbb{R}\}$ is a random process with power spectral density

$$S_X(\omega) = \frac{1}{(1+\omega^2)^2}$$

   Find the autocorrelation function of X_t.

2. Suppose that ω is a random variable with p.d.f. f_ω and θ is a random variable independent of ω uniformly distributed in $(-\pi, \pi)$. Define a random process by

$$X_t = a\cos(\omega t + \theta), \qquad t \in \mathbb{R}$$

   where a is a constant. Find the power spectral density of $\{X_t\}$.

3. Suppose events occur randomly in $T = [0, \infty)$ in the following way:

   (a) The numbers of events in nonoverlapping intervals are independent of one another.

   (b) $P(\text{exactly one event in } (t, t+\Delta t)) = \lambda(t)\Delta t + o(\Delta t)$, where $\lambda(t)$ is a continuous nonnegative function on $[0, \infty)$.

   (c) $P(\text{more than one event in an interval of length } \Delta t) = o(\Delta t)$.

   Define a random process $\{X_t, t \in T\}$ by $X_0 = 0$ and X_t is the number of events occurring in $(0, t]$.

   (a) For $t > s \geq 0$, show that $(X_t - X_s)$ is a Poisson random variable with parameter $\int_s^t \lambda(x)\, dx$.

   (b) Find the mean and autocorrelation functions of $\{X_t\}$.

4. Suppose that $\{X_t, t \in \mathbb{R}\}$ is a w.s.s., zero-mean, Gaussian random process with autocorrelation function $R_X(\tau), \tau \in \mathbb{R}$ and power spectral density $S_X(\omega), \omega \in \mathbb{R}$. Define the random process $\{Y_t, t \in \mathbb{R}\}$ by $Y_t = (X_t)^2, t \in \mathbb{R}$. Find the mean, autocorrelation, and power spectral density of $\{Y_t, t \in \mathbb{R}\}$.

5. Suppose U and V are independent random variables with $E[U] = E[V] = 0$ and $\mathrm{var}(U) = \mathrm{var}(V) = 1$. Define random processes by

$$X_t = U \cos t + V \sin T \qquad\qquad Y_t = U \sin t + V \cos t, \qquad t \in \mathbb{R}.$$

Find the autocorrelation and cross-correlation function s of $\{X_t, t \in \mathbb{R}\}$ and $\{Y_t, t \in \mathbb{R}\}$. Are $\{X_t\}$ and $\{Y_t\}$ jointly wide sense stationary? Are they individually wide sense stationary?