Utah State University
ECE 6010
Stochastic Processes
Homework # 8
Due Friday October 31, 2003

1. Suppose $\{X_t, t \geq 0\}$ is a homogeneous Poisson process with parameter λ. Define a random variable τ as the time of the first occurrence of an event. Find the p.d.f. and the mean of τ.

2. Suppose $\{X_t, t \in \mathbb{R}\}$ is a w.s.s. random process with autocorrelation function $R_X(\tau)$. Show that if R_X is continuous at $\tau = 0$ then it is continuous for all $\tau \in \mathbb{R}$. (Hint: Use the Schwartz inequality.)

3. Under the conditions of problem 2, show that for $a > 0$,
$$P(|X_{t+\tau} - X_t| \geq a) \leq \frac{2(R_X(0) - R_x(\tau))}{a^2}.$$

4. Suppose A and B are random variables with $E[A^2] < \infty$ and $E[B^2] < \infty$. Define the random processes $\{X_t, t \in \mathbb{R}\}$ and $\{Y_t, t \in \mathbb{R}\}$ by
$$X_t = A + Bt \qquad Y_t = B + At, \qquad t \in \mathbb{R}.$$
Find the mean, autocorrelation, and cross correlations of these random processes in terms of the moments of A and B.

5. Let $p_k(t, s) = X_t - X_s$, where X_t is a homoegenous Poisson counting process with rate λ. Show that the differential equation
$$\frac{\partial}{\partial t} p_k(t, s) = \lambda[p_{k-1}(t, s) - p_k(t, s)]$$
is solved by
$$p_k(t, s) = \frac{e^{-\lambda(t-s)}(\lambda(t-s))^k}{k!} \qquad k = 0, 1, \ldots, \qquad t > s \geq 0.$$

6. Let $p_k(t,s) = X_t - X_s$, where X_t is an inhomoegenous Poisson counting process with time-varyng rate λ_t. Show that the differential equation

$$\frac{\partial}{\partial t} p_k(t,s) = \lambda_t [p_{k-1}(t,s) - p_k(t,s)]$$

is solved by

$$p_k(t,s) = \frac{e^{-\lambda \int_s^t \lambda_x dx} (\int_s^t \lambda_x \, dx)^k}{k!} \qquad k = 0, 1, \ldots, \qquad t > s \geq 0.$$