Utah State University
ECE 6010
Stochastic Processes
Homework # 6
Due Friday October 28, 2005

1. Suppose $\{X_n\})_{n=1}^{\infty}$ is a sequence of independent r.v.s each of which is uniformly distributed on the interval $(0, 1)$. Define a sequence of r.v.s $\{Z_n\}$ by $Z_n = n(1-Y_n)$, where $Y_N = \max_{1\leq i\leq n} X_i$. Show that $\{Z_n\}_{n=1}^{\infty}$ converges in distribution to an exponential r.v. with p.d.f.
$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$
2. Suppose $X_n \to X$ (i.p.) and that there is a constant C such that $|X_n| \leq C$ for all n. Show that $X_n \to X$ (m.s.)
3. Suppose $X_n \to C$ (in distribution), where C is a constant. Show that $X_n \to C$ (i.p.)

Problems from Grimmet & Stirzaker

1. Exercise 7.2.1(b,c). On the converse, suppose X_n takes values ± 1 with probability 1/2.
2. Exercise 7.5.1.