

Utah State University  
ECE 6010  
Stochastic Processes  
Homework # 5  
Due Friday October 8, 2005

1. Box Muller: Let  $X_1 \sim \mathcal{U}(0, 1)$  and  $X_2 \sim \mathcal{U}(0, 1)$  (independent). Let

$$Y_1 = \sqrt{-2 \ln X_1} \cos 2\pi X_2 \quad Y_2 = \sqrt{-2 \ln X_1} \sin 2\pi X_2$$

Show that

$$Y_1 \sim \mathcal{N}(0, 1) \quad Y_2 \sim \mathcal{N}(0, 1).$$

2. If  $X$  and  $Y$  are independent and  $Y \sim \mathcal{U}(0, 1)$ , show that  $Z = X + Y$  has the density

$$f_Z(z) = F_X(z) - F_X(z - 1).$$

3. Let  $X \sim \mathcal{N}(0, \sigma^2)$  and  $Y \sim \mathcal{N}(0, \sigma^2)$  be independent. Let  $Z = X^2 + Y^2$ . Show that

$$f_Z(z) = \frac{1}{2\sigma^2} e^{-z/2\sigma^2} u(z)$$

Such a random variable is said to be chi-squared ( $\chi^2$ ) distributed with two degrees of freedom.

4. If  $X \in \mathcal{U}(0, 1)$  and  $Y = -(\ln(X))/\lambda$ , show that  $Y$  is exponentially distributed.  
5. Let  $g(x)$  be a monotone increasing function and let  $Y = g(X)$ . Show that

$$F_{XY}(x, y) = \begin{cases} F_X(x) & \text{if } y > g(x) \\ F_Y(y) & \text{if } y < g(x). \end{cases}$$

6. Let  $Z = aX + bY$ . Show that

$$f_Z(z) = \frac{1}{|a|} \int_{-\infty}^{\infty} f_{XY}\left(\frac{z - by}{a}, y\right) dy.$$

Hint: Let  $W = Y$  be an auxiliary variable.

7. Let  $X$  and  $Y$  be independent with

$$f_X(x) = \alpha e^{-\alpha x} u(x) \qquad f_Y(y) = \beta e^{-\beta y} u(y).$$

Determine the density of:

(a)  $Z = X/Y$ .

(b)  $Z = \max(X, Y)$ .

8. Show that if  $X, Y$  are i.i.d.  $\mathcal{N}(0, \sigma^2)$ , then  $Z = X/Y$  has a Cauchy distribution,

$$f_Z(z) = \frac{1/\pi}{z^2 + 1}.$$

Problems from Grimmet & Stirzaker.

1. Exercise 4.4.3.
2. Exercise 4.7.3.
3. Exercise 4.7.4.
4. Exercise 4.8.1 (Hint: Laplace transform)
5. Exercise 4.8.2 (Hint: Laplace transform)