# Utah State University
# ECE 6010
# Stochastic Processes
## Homework # 2
Due Friday Sept. 16, 2005

1. Suppose $X$ is a r.v. with c.d.f. $F_X$. Prove the following:

   (a) $F_X$ is nondecreasing.
   (b) $\lim_{a\to\infty} F_X(a) = 1$.
   (c) $\lim_{a\to-\infty} F_X(a) = 0$.
   (d) $F_X$ is right continuous.
   (e) $P(a < X \le b) = F_X(b) - F_X(a)$ if $b > a$.
   (f) $P(X = a) = F_X(a) - \lim_{b\to a^-} F_X(b)$.

   Also, find expressions for $P(a \le X \le b)$, $P(a \le X < b)$ and $P(a < X < b)$ in terms of $F_X$.

2. Show that the following are valid p.m.f.s:

   (a) Binomial: $f_X(a) = n!/((n-a)!a!)\pi^a(1-\pi)^{n-a}$ if $a \in \{0, 1, \dots, n\}$.
   (b) Poisson: $f_X(a) = e^{-\lambda}\lambda^a/a!$ for $a \in \{0, 1, \dots\}$.

3. Find the mean and variance of $X$ when $X$ is (a) $\mathcal{N}(\mu, \sigma^2)$; (b) Binomial$(n, \pi)$; (c) Poisson$(\lambda)$; (d) Exponential$(\lambda)$. Do not use characteristic functions.

4. Suppose that $X$ and $Y$ are jointly continuous. Show that

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y)\, dy \qquad x \in \mathbb{R}.$$

5. Suppose that $X$ and $Y$ are jointly Gaussian with parameters $\mu_x, \sigma_x^2, \mu_y, \sigma_y^2, \rho$. Show that $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$.

6. Suppose $X \sim \mathcal{N}(0, 1)$, and define $Y = X^2$. Are $X$ and $Y$ uncorrelelated? Are $X$ and $Y$ independent? Find the p.d.f. of $Y$. Are $X$ and $Y$ jointly continuous?

7. Show that $\operatorname{cov}(aX + b, cY + d) = ac \operatorname{cov}(X, Y)$.

8. Suppose $X \sim \mathcal{N}(0, \sigma^2)$. Use the ch.f. of $X$ to find an expression for $E[X^n]$, $n \in \mathbb{Z}^+$.

Problems from the Grimmett & Stirzaker text:

1. Ex 1.4.4
2. Ex 1.4.5. Hint: Let $C_i$ be the event that the $i$th door conceals the car, let $G$ be the event that you see a goat, and let $B$ be the event that you see Bill.
3. Ex 1.5.1.
4. Ex 1.5.2
5. Ex 1.5.7.
6. Prob. 1.8.5
7. Prob. 1.8.6.
8. Prob. 1.8.19.
9. Prob. 1.8.20.
10. Prob. 1.8.30.