

Utah State University
ECE 6010
Stochastic Processes
Homework #11
Due Friday Dec 10, 2004

These problems come from the Leon-Garcia text.

1. Let M_n denote the sequence of sample means from an i.i.d. random process X_n :

$$M_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

- (a) Is M_n a Markov process?
(b) If so, find the state transition p.m.f. $f_{M_n}(x|M_{n-1} = y)$
2. An urn initially contains five black balls and five white balls. The following experiment is repeated indefinitely. A ball is drawn from the urn; if the ball is white it is put back in the urn, otherwise it is left out. Let X_n be the number of black balls remaining in the urn after n draws from the urn.
- (a) Is X_n a Markov process? If so, find the appropriate transition probabilities.
(b) Do the transition probabilities depend on n ?
3. Let X_n be the Bernoulli i.i.d. process and let Y_n be

$$Y_n = X_n + X_{n-1}.$$

- (a) Show that Y_n is not a Markov process.
(b) Now consider the vector process $\mathbf{Z}_n = (X_n, X_{n-1})$. Show that \mathbf{Z}_n is a Markov process.
(c) Find the state transition diagram for \mathbf{Z}_n .
4. Show that the following autoregressive process is a Markov process:

$$Y_n = rY_{n-1} + X_n,$$

with $Y_n = 0$, where X_n is an i.i.d. process.

5. Let X_n be the Markov chain in problem 2.
- (a) Find the one-step transition probability matrix P for X_n .
(b) Find the two-step transition probability matrix P^2 . Check your answer by computing $p_{54}(2)$ and comparing it to the corresponding entry of P^2
(c) What happens to X_n as $n \rightarrow \infty$? Use your answer to guess the limit of P^n as $n \rightarrow \infty$.
6. Two players play the following game. A fair coin is flipped; if the outcome is heads, player A pays player B \$1, and if the outcome is tails, player B pays player A \$1. The game is continued until one of the players goes broke. Suppose initially that player A has \$1 and player B has \$2, so a total of \$3 is up for grabs. Let X_n be the number of dollars held by player A after n rounds.

- (a) Show that X_n is a Markov chain.
- (b) Draw the state transition diagram for X_n and give the one-step transition probability matrix P .
- (c) Use the state transition diagram to help you show that for n even,

$$p_{ii}(n) = (1/2)^n \quad i = 1, 2$$

$$p_{10}(n) = (2/3)(1 - (1/4)^{(n/2)}) = p_{23}(n)$$

- (d) Find the n -step transition probability matrix for n even using part c.
 - (e) Find the limit of P^n as $n \rightarrow \infty$.
 - (f) Find the probability that player A eventually wins.
7. A machine consists of two parts that fail and are repaired independently. A working part fails during any given day with probability a . A part that is not working is repaired by the next day with probability b . Let X_n be the number of working parts in day n .
- (a) Show that X_n is a three-state Markov chain and give its one-step transition probability matrix P .
 - (b) Show that the steady-state pmf $\boldsymbol{\pi}$ is binomial with parameter $p = b/(a + b)$
 - (c) What do you expect is the steady-state pmf for a machine that consists of n parts?