Utah State University
ECE 6010
Stochastic Processes
Homework #10
Due Friday November 21, 2003

1. Suppose $\{X_t, t \geq 0\}$ is a Wiener process. Define a process $\{Y_t, t \geq 0\}$ by

$$Y_t = X_{t+D} - X_t$$

for a fixed positive number D.

   (a) Find the mean and autocorrelation functions of $\{Y_t\}$.

   (b) Show that $\{Y_t\}$ is stationary and find its spectrum.

2. Suppose $\{X_t\}$ is $\{Y_t\}$ are zero mean and individually and jointly W.S.S. Show that the mean-square error associated with the noncausal Wiener filter for estimation of X_t from $\{Y_t, t \in \mathbb{R}\}$ is

$$\frac{1}{2\pi} \int -\infty^{\infty} [S_X(\omega) - \frac{|S_{XY}(\omega)|^2}{S_Y(\omega)}] \, d\omega.$$

3. Suppose $Y_t = S_t + N_t$ for $t \in \mathbb{R}$, where $\{S_t\}$ and $\{N_t\}$ are zero-mean, W.S.S., and orthogonal. Suppose that we wish to estimate

$$X_t = \int_{-\infty}^{\infty} k(t-\tau) S_\tau \, d\tau$$

with an estimate of the form

$$\hat{X}_t = \int_{-\infty}^{\infty} h(t-\tau) Y_\tau \, d\tau,$$

where k and h are impulse responses of linear time-invariant systems. Show that

$$E[(X_t - \hat{X}_t)^2] = \frac{1}{2\pi} \int_{-\infty}^{\infty} [|K(\omega) - H(\omega)|^2 S_S(\omega) + |H(\omega)|^2 S_N(\omega)] \, d\omega$$

where K and H are the transfer functions of k and h, respectively, and S_S and S_N are the power spectral densities of $\{S_t\}$ and $\{N_t\}$. (Note the case that $k(t) = \delta(t - \lambda)$ for some fixed $\lambda \in \mathbb{R}$.

4. Consider the situation of the previous problem with $k(t) = \delta(t - \lambda)$,

$$S_S(\omega) = \frac{A^2}{\alpha^2 + \omega^2} \qquad\qquad S_N(\omega) = \frac{N_0}{2}$$

(a) Find the noncausal Wiener filter for estimating X_t from $\{Y_t, t \in \mathbb{R}\}$. Find the corresponding mean-square error.

(b) Find the causal Wiener filter for estimating X_t from $\{Y_\tau, \tau \leq t\}$. Consider $\lambda < 0$ and $\lambda \geq 0$.