Utah State University
ECE 6010
Stochastic Processes
Homework # 1
Due Friday September 9.

Reading G&S, Chapter 1

Exercises

1. Create a list of all the stochastic processes you can think of that might occur in the real world (not just examples from the textbook). Be creative!

2. We defined a field to be a collection of sets that is closed under copmlementation and finite unions. Show that such a collection is also closed under finite intersections.

3. Using the axioms of probability, prove the following properties of probability:

   (a) $P(A^c) = 1 - P(A)$

   (b) $P(\emptyset) = 0$

   (c) $A \subset B \Rightarrow P(A) \leq P(B)$

   (d) $P(A \cup B) = P(A) + P(B) - P(AB)$

   (e) $A_1, A_2, \ldots \in \mathcal{F} \Rightarrow P(\cup_{i=1}^{\infty}) \leq \sum_{i=1}^{\infty} P(A_i)$

4. Suppose $P(B) > 0$. Prove the following properties of conditional probability:

   (a) $P(A|B) \geq 0$.

   (b) $P(\Omega|B) = 1$

   (c) For $A_1, A_2, \ldots \in \mathcal{F}$ with $A_iA_j = \emptyset$ for $i \neq j$, $P(\cup_{i=1}^{\infty} A_i|B) = \sum_{i=1}^{\infty} P(A_i|B)$

   (d) $AB = \emptyset \Rightarrow P(A|B) = 0$.

   (e) $P(B|B) = 1$

   (f) $A \subset B \Rightarrow P(A|B) \geq P(A)$

   (g) $B \subset A \Rightarrow P(A|B) = 1$.

5. Prove the law of total probability.

6. Prove Bayes rule
7. Suppose A and B are independent events. Show that A and B^c are also independent.