

Utah State University

ECE 3640

Homework # 11

Due Friday, April 21, 2006

- Reading: Chapter 5, Chapter 10

- Homework:

1. Problem 5.1-5
2. Problem 5.1-9
3. Problem 5.1-10
4. Problem 10.6-6
5. This exercise is to be in conjunction with MATLAB.

Let $x(t) = e^{-t}u(t)$. This is not a bandlimited signal. Let $x[n] = x(nT)$. In this problem you will explore the DFT for various combinations of sampling rates and zero padding.

- (a) Make a plot of $|X(\omega)|$ for $0 < f < 5$ Hz. (where $\omega = 2\pi f$.) For example,

```
freqrange = 0:.1:5; % make the range of frequency variables
Xf = 1 ./ sqrt((2*pi*freqrange).^2 + 1); % magnitude Fourier transform
plot(freqrange,Xf); % plot F.T.
```

You should overlay this plot over every spectrum computed below for comparison.

- (b) Let $T = 0.1$ be the sampling interval (i.e., $F_s = 10$). Let $N_0 = 128$, and sample 128 points of the function. Then plot the FFT of the sampled data:

```
N0=128; % number of sample points
T = 0.1; % sample interval
Fs = 1/T; % sample rate
k = (0:(N0-1)); % index range
k_2 = (0:(N0/2-1)); % half the points
x = T*exp(-(k*T)); % the sampled function. (Scale by T)
xout = fft(x,N0); % compute the FFT
plot(k_2*Fs/N0, abs(xout(1:(N0/2))), '-','r',freqrange, Xf, ':');
% plot sampled spectrum and true spectrum
```

% Note: the factor $fs/N0$ converts from bin number to frequency in Hz.

Print the plot. Comment on discrepancies between the true spectrum and the spectrum computed by the DFT.

- (c) Now try some variations.
- i. Take 128 point of data, with zero-pad to 512 points.
 - ii. Take 128 points of data, with $T = 0.5$.
 - iii. Take 128 points of data, with $T = 0.01$.
 - iv. Take 512 points of data, with $T = 0.1$
 - v. Take 512 points of data, with $T = 0.5$

Turn in appropriate plots for each variation. Interpret all the results in light of aliasing and spectral leakage.

6. Cyclic convolution. Let

$$x[k] = \begin{cases} 1 & 0 \leq k \leq 16 \\ 0 & \text{otherwise} \end{cases}$$

and let

$$y[k] = \begin{cases} k & 0 \leq k \leq 8 \\ 16 - k & 8 \leq k \leq 16 \\ 0 & \text{otherwise} \end{cases}$$

(a) Using MATLAB, find the linear convolution of x and y :

```
x = ones(1,16); % set up x data
y = [1 2 3 4 5 6 7 8 7 6 5 4 3 2 1]; % set up y data
z = conv(x,y); % convolve
stem(0:length(z)-1,z); % plot the convolved results
```

(b) Now test the convolution theorem by taking transforms of the signals:

```
X = fft(x,32); % take the DFT of x data
Y = fft(y,32); % take the DFT of y data
Z = X .* Y; % Multiply. .* is point-for-point multiply
znew = real(ifft(Z,32)); % compute the inverse transform
% use real() to get rid of the imaginary part
% (which should be very small)
stem(0:length(znew)-1,znew); % plot the convolution.
```

(c) Compare the plot from part (a) with the plot from part (b).

(d) Now we will examine cyclical convolution by dealing with 16 point FFTs.

```
X1 = fft(x,16);
Y1 = fft(y,16);
Z1 = X1 .* Y1;
z1 = ifft(Z1,16);
stem(0:15, real(z1));
```

Using cyclical convolution, account for this strange result.