ECE 7680
Homework Assignments

#1 Due: Jan 18, 2000

- Reading:
  1. Chapter 1 of C&T. You may not understand all of the notation yet, but it will give you a good overview.
  2. Chapter 2 of C&T. (Don't worry about section 2.10.)
  3. Begin *Silicon Dreams*.
  4. pp. 23–29, 41–82 of Mackay (you don't need to exhaustively read all the solutions, but many of them are interesting.
- Problems:
  1. Problems 1, 2, 3, and 5 from chapter 2.
  2. Go to the library and look through some *IEEE Transactions on Information Theory*. Peruse a few to get an idea of what IT is all about. (You won't even understand most of the titles, but look anyway.) Begin thinking about a topic for the term project.

 # 2 Due: Jan 25, 2000

- Reading: More on *Silicon Dreams*
- Problems:
  1. Problem 4.7 from Proakis & Salehi
  2. Problem 4.8 from Proakis & Salehi
  3. Problem 4.11 from Proakis & Salehi
  4. Problem 4.15 from Proakis & Salehi
  5. Problem 4.29 from Proakis & Salehi
  6. Problem 4.31 from Proakis & Salehi
  7. Problem 4.35 from Proakis & Salehi. The differential entropy of a continuous random variable is defined as
     $$H(X) = -\int_{-\infty}^{\infty} f_X(x) log f_X(x)\, dx,$$
     where $f_X(x)$ is the p.d.f. of X.
  8. Problem 4.38 from Proakis & Salehi
  9. Problem 4.39 from Proakis & Salehi

#3 Due: Feb 1, 2000.

- Reading: More on *Silicon Dreams*
- Problems:
  1. Show by constrained optimization that the uniform distribution maximizes the entropy of a discrete random variable.
  2. Problems 2.8, 2.9, 2.12, 2.13(a), 2.14, 2.16, 2.17, 2.19, 2.20, 2.24

3 Due: Feb 9, 2000

- Reading:
  1. More on *Silicon Dreams*
  2. Paper by Bell & Sejnowski
  3. Section 14.8 of Moon & Stirling
  4. Appendix E of Moon & Stirling
- Problems:
  1. Given $f_X(x)$, determine a transformation g as in $Y = g(X)$ so that $Y \sim \mathcal{U}(0, 1)$.
  2. Given that $X \sim \mathcal{U}(0, 1)$, determine a transformation g as in $Y = g(X)$ so that $f_Y(y)$ has some desired form.
  3. Scalar case: If $y = g(x) = \frac{1}{1+e^{-u}}$, where $u = wx + w_0$, show that

     $$\frac{\partial y}{\partial x} = wy(1 - y)$$

     and that

     $$\frac{\partial}{\partial w}\frac{\partial y}{\partial x} = y(1 - y)[1 + wx(1 - 2y)]$$

     and that

     $$\frac{\partial}{\partial w_0}\frac{\partial y}{\partial x} = wy(1 - y)x(1 - 2y).$$

     Hence, explain the learning rule

     $$\Delta w \propto \frac{1}{w} + x(1 - 2y)$$

     and

     $$\Delta w_0 \propto 1 - 2y.$$

  4. If $y = g(x) = \tanh(wx + w_0)$, show that $\Delta w \propto \frac{1}{w} - 2xy$.
  5. When $\mathbf{y} = g(W\mathbf{x} + \mathbf{w}_0)$, where g is the logistic function

     $$g(u) = \frac{1}{1 + e^{-u}},$$

     applied element-by-element, show that

     $$\Delta W \propto [W^{-T}] + (\mathbf{1} - 2\mathbf{y})xbf^T$$

     and

     $$\Delta \mathbf{w}_0 \propto \mathbf{1} - 2\mathbf{y}.$$

6. For $\mathbf{x} = (x_1, \ldots, x_n)$, define

$$I(\mathbf{x}) = \int p(\mathbf{x}) \log \frac{p(\mathbf{x})}{\prod_{i=1}^n p(x_i)} d\mathbf{x} = D(p(\mathbf{x}) || \prod_{i=1}^n p(x_i)).$$

Show that

$$H(\mathbf{x}) = \sum_{i=1}^n H(x_i) - I(\mathbf{x}).$$

This is a generalization of the formula (2.45) of the text.

7. Show that:

$$\frac{\partial \det(X)}{\partial X} = \begin{cases} \det(X) X^{-T} & \text{general } X \\ \det(X)(2X^{-1} - \mathrm{diag}(X^1)) & \text{symmetric } X \end{cases}$$

$$\frac{\partial \log \det(X)}{\partial X} = \frac{1}{\det(X)} \frac{\partial \det(X)}{\partial X}$$

Hint: section E.2.

HW # 4 Due Feb 16, 2000

- Reading:
  1. Read chapter 3
  2. Finish *Silicon Dreams*.
- Problems:
  1. Give an estimate of the number of binary sequences of length 10,000 with 3000 zeros and 7000 ones. Use both an information-theoretic and a combinatoric argument, and show that they are similar by means of Stirlings relation:

  $$n! \approx e^{-n} n^n \sqrt{2\pi n}$$

  2. Problem 3.1, 3.2, 3.3, 3.5, 3.6