Reading

- Chapter 2 in the Anderson text

Problems

1. Let the transmitter pulse $v(t)$ be the NRZ pulse given by

$$v(t) = \begin{cases} 1 & -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

   The linear receiver applies a filter with impulse response $v(-t)$ and then samples the output at multiples of T.

   (a) Draw an eye diagram corresponding to the signal at the input to the linear receiver.

   (b) Draw an eye diagram corresponding to the signal at the output of the linear receiver.

   (c) Based on these eye diagrams, explain why the NRZ pulse is both a Nyquist pulse and a square-root Nyquist pulse.

2. (a) Express the three waveforms in Figure 2.23 (page 46 of the Anderson text) in terms of an orthonormal basis. To do this problem, apply the Gram-Schmidt procedure outlined on pages 44-45 of the text.

   (b) Draw the vectors in signal space. Your drawing should look something like Figure 2.25 (page 47).

   (c) What is the dimension of signal space in this problem?

   (d) Draw the orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$.

3. Let $\phi_1(t), \cdots, \phi_J(t)$ be a set of J orthonormal functions defined over the interval $0 \leq t \leq T$. This means that

$$\langle \phi_i(t), \phi_j(t) \rangle = \int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

   Let $r(t)$ be a signal that is not in span $\{\phi_1(t), \cdots, \phi_M(t)\}$. Let $\hat{r}(t) = \sum_{j=1}^{J} c_j \phi_j(t)$ be a signal that is in span $\{\phi_1(t), \cdots, \phi_M(t)\}$ that is an approximation of $r(t)$. Define the error

$$e(t) = r(t) - \hat{r}(t) = r(t) - \sum_{j=1}^{J} c_j \phi_j(t)$$

   Prove that the coefficients $c_1, \cdots, c_M$ that minimize the energy in $e(t)$

$$E_{e(t)} = \int_0^T e^2(t)dt$$

   are given by

$$c_j = \langle r(t), \phi_j(t) \rangle$$

4. Let $\phi_1(t), \cdots, \phi_J(t)$ be a set of J orthonormal function defined over the interval $0 \leq t \leq T$. Let $s(t)$ be a function in the span of these orthonormal functions.

$$s(t) = \sum_{j=1}^{J} c_j \phi_j(t)$$

Collect the coefficients c_i into a vector

$$\mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_J \end{bmatrix}$$

Prove that the energy in $s(t)$ is related to the norm (length) of the vector $\mathbf{c}$ as follows

$$E_{e(t)} = \int_0^T e^2(t)dt = \sum_{j=1}^{J} c_j^2 = \|\mathbf{c}\|^2$$

5. Consider the likelihoods $P(B|A)$ and prior probabilities $P(A)$ tabulated below.Compute (using Bayes rule) the posterior probabilities $P(A = i|B = j)$ for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$.

$P(B = i \vert A = j)$	$j = 1$	$j = 2$	$j = 3$
$i = 1$	0.1	0.5	0.8
$i = 2$	0.3	0.1	0.0
$i = 3$	0.4	0.1	0.1
$i = 4$	0.2	0.3	0.1

i	$P(A = i)$
1	0.3
2	0.6
3	0.1