Reading

- Chapter 2 in the Anderson text

Problems

1. Prove Nyquist's First Criterion (Theorem 2.2-1 on page 19 of the Anderson text). Note that if you follow the proof given in the book, then you must first prove the Poisson sum formula. Here is a hint.

   Multiply a function $v(t)$ by an impulse train as follows.

   $$v(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} v(nT)\delta(t - nT)$$

   Now compute the continuous-time Fourier transform of both sides. Note that the left hand side is a product of two functions (use the modulation property–multiplication in time corresponds to convolution in the frequency domain). The $\delta(t - nT)$ on the right hand side are the only functions of of t. If all goes well, then you end up with the Poisson sum formula.

   $$\sum_{n=-\infty}^{\infty} V\left(f - \frac{n}{T}\right) = T \sum_{m=-\infty}^{\infty} v(mT)\, e^{-j2\pi mTf}$$

   Now you can follow the book to complete the proof.

2. Compute the Fourier transforms of the following:

   (a) unit-energy NRZ pulse

   (b) unit-energy RZ pulse

   (c) unit-energy MAN pulse

   (d) unit-energy HS pulse which is given by

   $$v_{\text{HS}}(t) = \begin{cases} \frac{2}{T}\sin\left(\frac{\pi t}{T}\right) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

3. Let $V(f)$ be the Fourier transform of $v(t)$ given by

   $$V(f) = \begin{cases} 1 & |f| < 100 \text{ Hz} \\ 2 - \frac{|f|}{100} & 100 < |f| < 200 \\ 0 & \text{otherwise} \end{cases}$$

   (a) For what symbol rate is $v(t)$ a Nyquist pulse?

   (b) At this symbol rate, what is the excess bandwidth of the pulse?

   (c) Sketch $V(f), V(f-(1/T)), V(f+(1/T)), V(f-(2/T)), V(f+(2/T))$ terms of the sum in Equation 2.2-2 (page 19 of the Anderson text) and show that the Nyquist criterion is satisfied.

4. Let $v(t)$ be a pulse with odd symmetry, $v(t) = -v(-t)$. Can $v(t)$ be a Nyquist pulse?

5. Consider a digital modulator that transmits 3 bits per symbol and uses the unit-energy HS pulse for transmission.

   (a) Construct a look up table with three columns showing the correspondence between: bits, symbols, and the modulated pulses.

   (b) Sketch the modulator, the channel, and a sampling receiver.

   (c) For the bit sequence {001 100 111 011}, sketch the signal or sequence at each point in the modulator/channel/receiver diagram.

6. Intersymbol interference (ISI)

   (a) Let $v(t)$ be the following pulse,

$$v(t) = \begin{cases} 1 & -\frac{3T}{2} \leq t \leq \frac{5T}{2} \\ 0 & \text{otherwise} \end{cases}$$

   What is the ISI?

   (b) Let $v(t)$ be the following pulse,

$$v(t) = \text{sinc}\left(\frac{t}{2T}\right) = \frac{\sin\left(\frac{\pi t}{2T}\right)}{\frac{\pi t}{2T}}$$

   What is the ISI?