Homework 10

Reading

- Chapter 2 in the Anderson text

Problems

1. Suppose $\phi_1(t)$ and $\phi_2(t)$ be an orthonormal basis for signal space given by

$$\phi_1(t) = \begin{cases} \sqrt{\frac{2}{T}} & 0 \le t < \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_2(t) = \begin{cases} \sqrt{\frac{2}{T}} & \frac{T}{2} \le t < T \\ 0 & \text{otherwise} \end{cases}$$

   (a) Verify that $\phi_1(t)$ and $\phi_2(t)$ are orthonormal.

   (b) Sketch matched filters for $\phi_1(t)$ and $\phi_2(t)$.

   (c) The following look-up table is used in the modulator.

signal	bits (b_1, b_2, b_3)	symbol (s_{m1}, s_{m2})
$\mathbf{s}_1$	000	$(+1, +3)$
$\mathbf{s}_2$	001	$(+1, +1)$
$\mathbf{s}_3$	010	$(+1, -1)$
$\mathbf{s}_4$	011	$(+1, -3)$
$\mathbf{s}_5$	100	$(-1, +3)$
$\mathbf{s}_6$	101	$(-1, +1)$
$\mathbf{s}_7$	110	$(-1, -1)$
$\mathbf{s}_8$	111	$(-1, -3)$

   Sketch the eight possible transmitted waveforms. Assume that the symbol period is $T = 1$ second.

   (d) Sketch the waveform that is transmitted if the six bits (100 010) are fed into the modulator.

   (e) Sketch the constellation.

   (f) Sketch the ML decision regions.

   (g) What is $D_{\min}$?

   (h) Compute the average symbol and bit energy.

   (i) Suppose that the following set of measurements were observed at the output of a bank of filters matched to $\phi_1(t)$ and $\phi_2(t)$.

Symbol Period	$n = 0$	$n = 1$	$n = 2$	$n = 3$
$\mathbf{r} = (r_1, r_2)$	$(5.1, -1.6)$	$(-2.1, -2.1)$	$(-0.7, 1.3)$	$(0.2, 3.5)$

   What what symbol sequence and bit sequence would come out of the ML receiver for this set of measurements?

   (j) Compute the probability that the ML receiver decides that $\mathbf{s}_5$ was transmitted when in fact $\mathbf{s}_1$ was actually transmitted.

   (k) Use the union bound to compute an upper bound for the probability of error for this constellation using an ML receiver.

2. Sketch the impulse response of a filter matched to the square-root raised cosine pulse. Follow the procedure on page 60 and 61 of the Anderson text.