ECE 5660
Spring 2006

Utah State University

Homework 4

Due: Wednesday, February 8, 2005

Problems

1. A real-valued signal $x(t)$ is known to be uniquely determined by its samples when the sampling frequency is $\omega_s = 2\pi \cdot 5000$ rads/sec. For what values of ω is $X(j\omega)$ guaranteed to be zero?

2. A continuous-time signal $x(t)$ is obtained at the output of an ideal low pass filter with cutoff frequency $\omega_o = 2\pi \cdot 500$ rads/sec. If impulse-train sampling is performed on $x(t)$, which of the following sampling periods would guarantee that $x(t)$ can be recovered from its sampled version using an appropriate low pass filter?

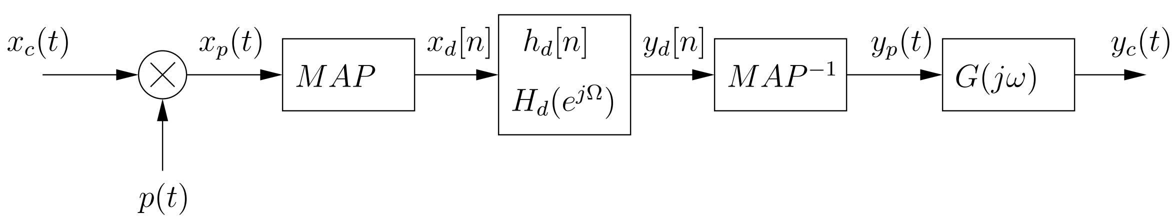
   (a) $T = 0.5 \times 10^{-3}$
   (b) $T = 2 \times 10^{-3}$
   (c) $T = 10^{-4}$

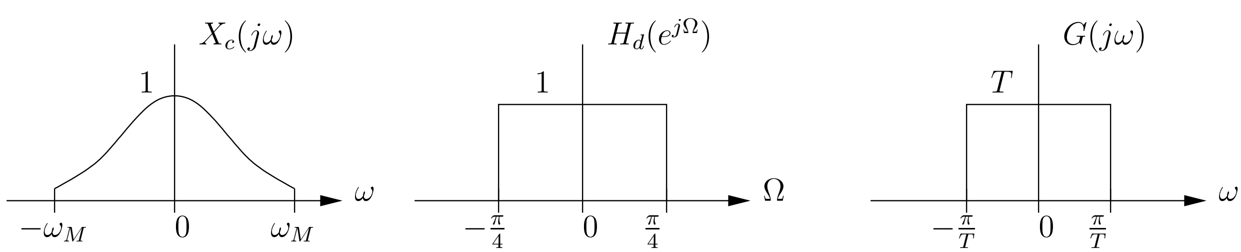
3. Suppose the sinusoidal signal $x(t) = \cos(2\pi f t)$ is sampled at the rate f_s Hz. Given the continuous-time frequency f and the sample rate f_s, compute the discrete-time frequency after aliasing.

   (a) $f = 2100$ Hz, $f_s = 1600$ Hz.
   (b) $f = -2100$ Hz, $f_s = 1600$ Hz.
   (c) $f = 1500$ Hz, $f_s = 1600$ Hz.
   (d) $f = -4500$ Hz, $f_s = 1600$ Hz.
   (e) Show that $\cos([\pi + \epsilon]n) = \cos([\pi - \epsilon]n)$ for all n where ϵ is a small positive number.

4. A real-valued discrete-time signal $x[n]$ has a Fourier transform $X(e^{j\Omega})$ that is zero for $\frac{3\pi}{14} \leq |\Omega| \leq \pi$. This signal can be up sampled by a factor of L and and then down sampled by a factor of M to give a signal $x_r[n]$ for which the spectrum $X_r(e^{j\Omega})$ occupies the entire region $|\Omega| \leq \pi$. Specify the values of L and M and sketch the spectra of the signal at each point in the up sampling and down sampling process.

5. The figure below shows the overall system for filtering a continuous-time signal using a discrete-time filter where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ and T is the sampling period. If $X_c(j\omega)$, $H(e^{j\Omega})$ and $G(j\omega)$ are as shown in the figure with $\omega_M = 2\pi \cdot 5000$ rads/sec and $\frac{1}{T} = 20$ kHz, sketch $X_p(j\omega)$, $X_d(e^{j\Omega})$, $Y_d(e^{j\Omega})$, $Y_p(j\omega)$, and $Y_c(j\omega)$. The MAP function converts and impulse train to a discrete-time sequence and MAP^{-1} does the opposite.





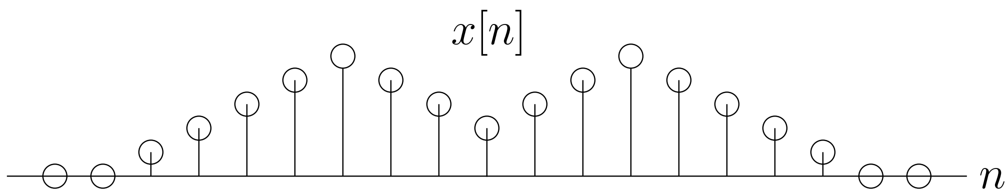
6. *(Downsampling by a factor of 2)* Consider a discrete-time sequence $x[n]$ from which we form two new sequences, $x_p[n]$ and $x_d[n]$, where $x_p[n]$ corresponds to multiplying $x[n]$ by $p[n] = \sum_{k=-\infty}^{\infty} \delta[n - kN]$ and $N = 2$, and $x_d[n]$ is equal to $x_p[nN]$ and $N = 2$. That is, $x_d[n]$ is equal every N^{th} sample of $x_p[n]$.

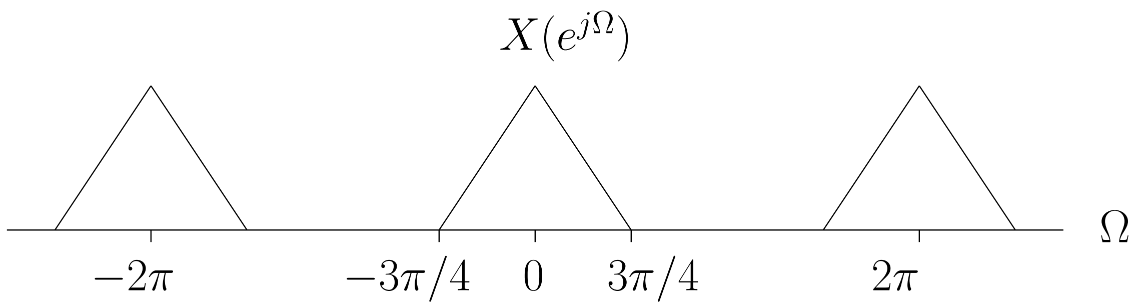
$$x_p[n] = \begin{cases} x[n] & n = 0, \pm 2, \pm 4, \cdots \\ 0 & n = \pm 1, \pm 3, \cdots \end{cases}$$

$$x_d[n] = x_p[nN] = x[nN] \qquad\qquad N = 2$$

(a) If $x[n]$ is the sequence illustrated below, then sketch $x_p[n]$ and $x_d[n]$.

(b) If $X(e^{j\Omega})$ is as shown below, sketch $X_p(e^{j\Omega})$ and $X_d(e^{j\Omega})$.





7. *(Upsampling by a factor of 2)* Consider a discrete-time sequence $x[n]$ from which we form two new sequences $x_p[n]$ and $x_u[n]$. The sequence $x_p[n]$ corresponds to $x[n]$ with one zero sample inserted between each sample of $x[n]$. The sequence $x_u[n]$ is a low-pass filtered version of $x_p[n]$ using an ideal low-pass filter with cut off frequency of $\pi/8$.

$$x_p[n] = \begin{cases} x[n/2] & \text{if } n/2 \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}$$

$$x_u[n] = h[n] * x_p[n]$$

(a) If $x[n]$ is the sequence illustrated below, then sketch $x_p[n]$ and $x_u[n]$.

(b) If $X(e^{j\Omega})$ is as shown below, sketch $X_p(e^{j\Omega})$ and $X_u(e^{j\Omega})$.