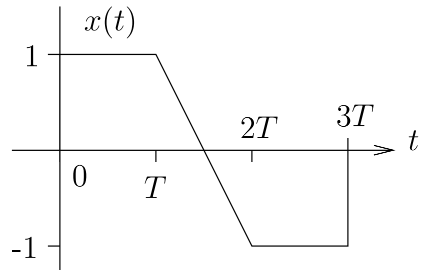
Due: Wednesday, February 1, 2006

Problems

1. Let the signal $x(t)$ shown below be input to a phase modulator producing $y(t) = \cos(\theta(t))$.



Assume the carrier frequency is $\omega_c = \frac{4\pi}{T}$ rads/sec, the carrier phase $\theta_0 = 0$ and that $k_p = 1$.

(a) Sketch $\theta(t)$.

(b) Sketch the instantaneous frequency $\omega_i(t)$.

(c) Sketch $y(t)$.

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2. Let the signal $x(t)$ shown in Problem 1 be input to a frequency modulator producing $y(t) = \cos(\theta(t))$. Assume the carrier frequency is $\omega_c = \frac{4\pi}{T}$ rads/sec, the carrier phase $\theta_0 = 0$ and that $k_p = 1$.

(a) Sketch $\theta(t)$.

(b) Sketch the instantaneous frequency $\omega_i(t)$.

(c) Sketch $y(t)$.

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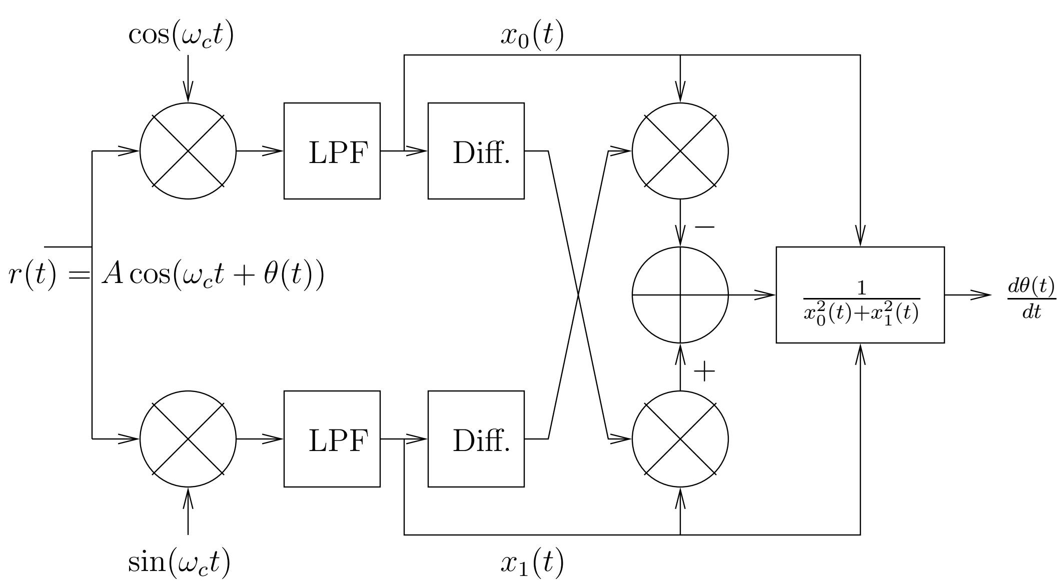
3. Show that the output of the system shown below is $\frac{d\theta(t)}{dt}$. If $r(t) = A\cos(\omega_c t + \theta(t))$ is the output of an FM modulator with input signal $x(t)$, show how $\frac{d\theta(t)}{dt}$ is related to $x(t)$.



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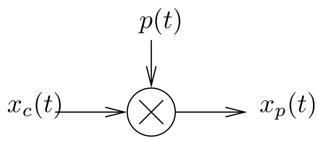
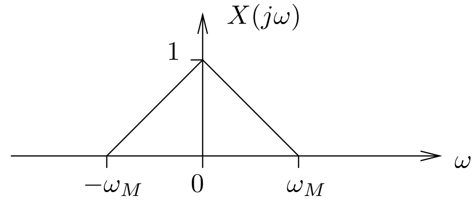
4. Let $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ in the figure below.





(a) Derive an expression for $x_p(t)$ in terms of $x_c(t)$. Simplify as far as possible using the properties of $\delta(t)$.

(b) Using the modulation property of the Fourier transform, derive the relationship between $X_p(j\omega)$ and $X_c(j\omega)$.

(c) With the spectrum $X_c(j\omega)$ given above, sketch $X_p(j\omega)$ assuming $\frac{2\pi}{T} > 2\omega_M$.

(d) With the spectrum $X_c(j\omega)$ given above, sketch $X_p(j\omega)$ assuming $\omega_M < \frac{2\pi}{T} < 2\omega_M$.

(e) Design an ideal low pass filter $H(j\omega)$ to recover $x_c(t)$ from $x_p(t)$ assuming $\frac{2\pi}{T} > 2\omega_M$. Illustrate with a sketch the operation of $H(j\omega)$ in the time and frequency domains.

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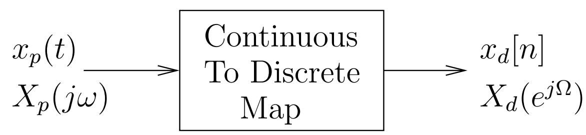
5. (a) Derive an expression for $X_d(e^{j\Omega})$ in terms of $X_p(j\omega)$ for the continuous to discrete block shown below.



(b) Using the expression for $X_p(j\omega)$ derived in Problem 4, show how $X_d(e^{j\Omega})$ is related to $X_c(j\omega)$.

(c) For the spectrum shown in Problem 4, sketch $X_d(e^{j\Omega})$ assuming $\frac{2\pi}{T} > 2\omega_M$.