Due: Wednesday, January 25, 2006

Reading

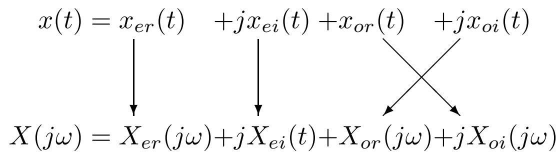
- Read pages 277-300 in Lathi.

Problems

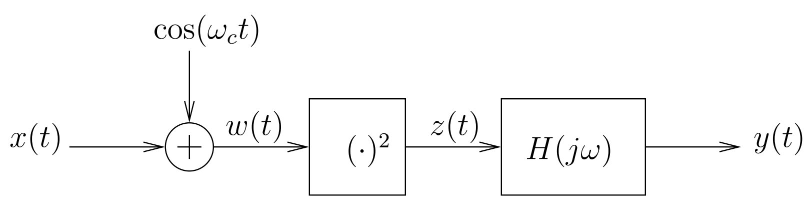
1. A complex signal $x(t)$ can be decomposed into four components

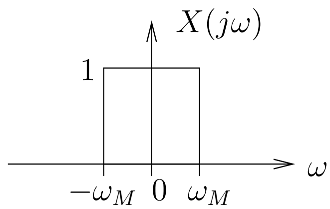
$$x(t) = x_{er}(t) + x_{or}(t) + jx_{ei}(t) + jx_{oi}(t)$$

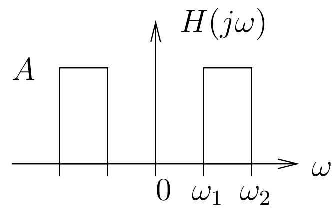
in which said components are the real-even, real-odd, imaginary-even, and imaginary-odd parts of $x(t)$, respectively. The same decomposition can be performed on the Fourier transform $X(j\omega)$ of $x(t)$. The functions $x_{er}(t)$ and $x_{ei}(t)$ even functions of t, where as $x_{or}(t)$ and $x_{oi}(t)$ are odd. Show that the components of $x(t)$ and $X(j\omega)$ transform as in the diagram below.



2. An amplitude modulator is a device that accepts two inputs $x(t)$ and $c(t)$ and generates the product of the two signals on the output $y(t) = x(t)c(t)$. In practice, an analog multiplier may be difficult to build. Therefore, practical systems often use a nonlinear processing element and additional processing to produce the same effect as a multiplication. For example, consider the following system that uses a squaring nonlinearity.







The spectrum of $x(t)$ and response of $H(j\omega)$ are also shown. The filter $H(j\omega)$ is an ideal band pass filter with response,

$$H(j\omega) = \begin{cases} A & \omega_1 \le |\omega| \le \omega_2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the spectra $W(j\omega)$ and $Z(j\omega)$ of $w(t)$ and $z(t)$.

(b) Derive a mathematical expression for $z(t)$ in terms of $x(t)$ and $\cos(\omega_c t)$ and simplify as far as possible using trigonometric identities.

(c) Determine the parameters A, ω_1, ω_2 of $H(j\omega)$ so that $y(t)$ is an amplitude modulated version of $x(t)$, $y(t) = x(t)\cos(\omega_c t)$.

(d) Sketch the spectrum $Y(j\omega)$ of $y(t)$.

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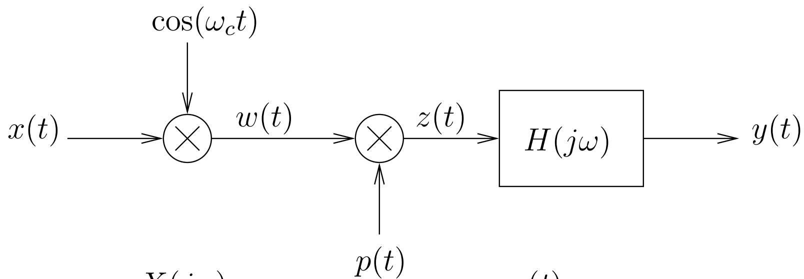
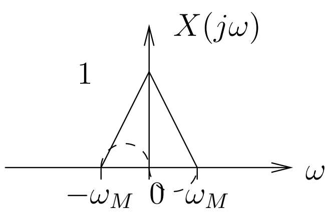
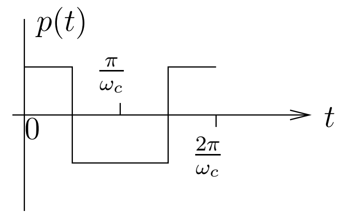
3. Consider the situation depicted in the figure below in which an AM signal is generated by modulating $x(t)$ by $\cos(\omega_c t)$. Instead of demodulating using $\cos(\omega_c t)$, this problem explores demodulation with a square wave $p(t)$ that has the same zero crossings as $\cos(\omega_c t)$. The spectrum of $X(j\omega)$ and one period of the square wave are shown below.







(a) Sketch the spectra $W(j\omega), P(j\omega)$, and $Z(j\omega)$ of $w(t), p(t)$, and $z(t)$.

(b) Determine the gain and pass and stop band edges of the low pass filter $H(j\omega)$ so that $y(t) = x(t)$. Sketch the filter response.

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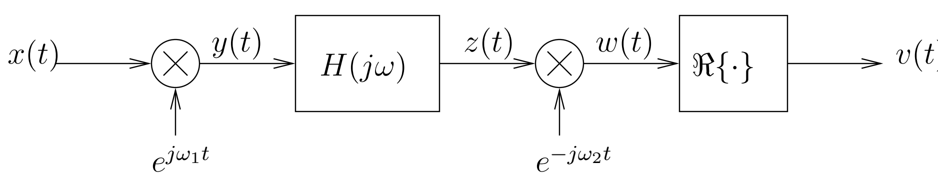
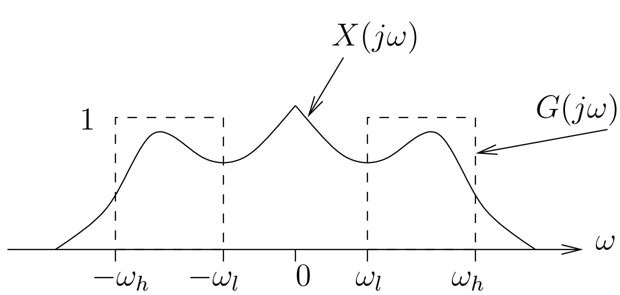
4. The goal of this problem is to design a bandpass filter $G(j\omega)$ using a low pass filter $H(j\omega)$ and complex exponential modulators as depicted in the figure below where $H(j\omega)$ is a low pass filter with response given by

$$H(j\omega) = \begin{cases} A & |\omega| \le \omega_0 \\ 0 & \text{otherwise} \end{cases}.$$



The spectrum of $X(j\omega)$ and the desired band pass filter response are shown below.



(a) Determine ω_1, ω_2 and the low pass filter parameters A, ω_0 so that $v(t)$ is a band pass filtered version of $x(t)$.

(b) Sketch the spectra $Y(j\omega), Z(j\omega), W(j\omega), V(j\omega)$ of $y(t), z(t), w(t), v(t)$ and the response of $H(j\omega)$ for the parameters you determined in part 4a.

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5. Derive the Fourier transform relationship,

$$U(j\omega) = \begin{cases} 0 & \omega < 0 \\ 2 & \omega > 0 \end{cases} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad u(t) = \delta(t) + j\frac{1}{\pi t}$$

using the Fourier transform property,

$$j\frac{\partial V(j\omega)}{\partial \omega} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad t \cdot v(t).$$

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6. Sketch a signal processing system with real input $x(t)$, transfer function $U(j\omega)$ given in problem 5, and output $y(t)$.

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7. Single side band systems.

   (a) Sketch a signal processing system for generating a single side band/lower side band modulation system. Use ω_c as the carrier frequency. You may use only real multipliers.

   (b) Sketch the spectrum of signals at each point in the system. For the input signal spectrum $X(j\omega)$ use the spectrum given in problem 3.

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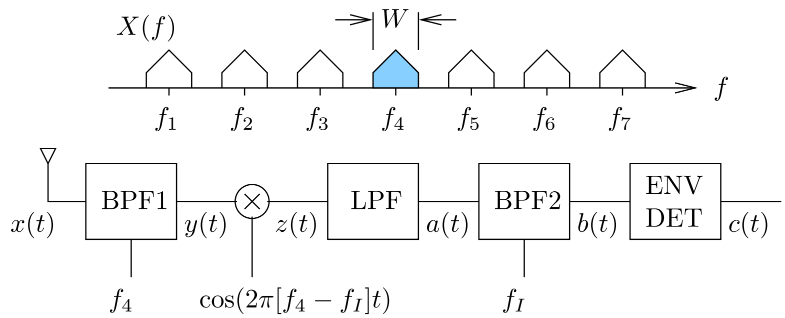
8. Demodulating single side band signals.

   (a) Sketch a system that will demodulate a single side band/upper side band modulated signal. The carrier frequency is ω_c. You may use only real multipliers.

   (b) Sketch the spectrum of signals at each point in the system. For the modulated signal spectrum, assume $X(j\omega)$ as given in problem 3 was modulated using single side band/upper side band modulation.

9. Intermediate frequency.



The figure above shows a portion of the frequency spectrum in which seven signals are frequency division multiplexed. The center frequencies are f_i for $i = 1, 2, \cdots, 7$. The each signal has bandwidth $W = 10$ kHz and the channel spacing is $f_i - f_{i-1} = 30$ kHz. The goal is to separate the shaded signal from the others and demodulate it using the signal processing system shown in the block diagram above.

The block diagram consists of three filters, a modulator, and an envelope detector. The filter BPF1 pass band covers the range $|f - f_4| < 40$ kHz and the stop band covers $|f - f_4| > 75$ kHz. Assume the pass band gain is 1. Let $f_4 = 1$ MHz and $f_I = 200$ kHz.

Design LPF, BPF2, and ENV DET so that $c(t)$ is a demodulated version of the signal modulated at f_4. To design the low pass filter (LPF), choose the pass band and stop band edge frequencies and the pass band gain. Assume the pass band gain is 1. To design the band pass filter (BPF2), choose the pass band and stop band edge frequencies and let the pass band gain be 1. Remember that the pass band of BPF2 is centered on f_I. For the filters, use the widest possible transition bands between pass and stop bands. To design the envelope detector (ENV DET), specify the RC constant.