Let $X$ be the time required to complete a test with a maximum completion time of 2 hours. The data used in the analysis is collected every 15 minutes, resulting in the following table of probabilities for values of $x = 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(X=x)$</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Plot the probability mass function ($pmf$) for $X$.

(b) Calculate a table of the cumulative distribution function (CDF) for $X$.

(c) Plot the cumulative distribution function (CDF) for $X$.

Calculate the following probabilities:

(d) $P(X = 1.25)$  
(e) $P(X \leq 1.50)$  
(f) $P(X < 1.50)$  
(g) $P(X \leq 0.75)$

(h) $P(X > 1.00)$  
(i) $P(X \geq 1.00)$  
(j) $P(0.75 \leq X \leq 1.75)$  
(k) $P(0.75 < X \leq 1.75)$

(l) $P(0.75 \leq X < 1.75)$  
(m) $P(0.75 < X < 1.75)$  
(n) $P(1 < X < 1.25)$  
(o) $P(0.8 < X < 1.65)$

(n) Calculate the expected value, or mean, of $X$.

(o) Calculate the variance of $X$.

(p) Calculate the standard deviation of $X$.

(q) Calculate the skewness of $X$.

(r) Calculate the kurtosis of $X$.

Let $Y = g(X) = 20X^2$ represent the cost, in dollars, of monitoring the test for $X$ hours. What is the expected value of $Y$?

(t) What is the variance of $Y$?

Let $X$ represent the number of defects per square inch found in a photographic film while calibrating a new film-producing machine. The table below shows that cumulative distribution function for $X$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$F(x)$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Calculate a table of the probability mass function ($pmf$) for $X$.

(b) Plot the probability mass function ($pmf$) for $X$.

(c) Plot the cumulative distribution function (CDF) for $X$.

Calculate the following probabilities:

(d) $P(X = 2)$  
(e) $P(X \leq 4)$  
(f) $P(X < 4)$  
(g) $P(X \leq 2)$

(h) $P(X > 2)$  
(i) $P(X \geq 2)$  
(j) $P(1 \leq X \leq 4)$  
(k) $P(1 < X \leq 4)$

(l) $P(1 \leq X < 4)$  
(m) $P(1 < X < 4)$  
(n) $P(1.5 < X < 4.5)$  
(o) $P(3 < X < 4)$

(n) Calculate the expected value, or mean, of $X$.

(o) Calculate the variance of $X$.

(p) Calculate the standard deviation of $X$.

(q) Calculate the skewness of $X$.

(r) Calculate the kurtosis of $X$. 
Let $X$ represent the number of droplets per cubic inch detected in the air over an arid region by using a Doppler radar measurement. It is suggested that the probability mass function of $X$ is given by

$$f(x) = \begin{cases} \frac{c}{x^2}, & \text{for } x = 0, 1, \ldots, 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) Calculate the value of $c$ so that $f(x)$ is a proper probability mass function (pmf) for $X$.
(b) Plot the probability mass function (pmf) for $X$.
(c) Plot the cumulative distribution function (CDF) for $X$.

Calculate the following probabilities:

(d) $P(X = 3)$
(e) $P(X \leq 4)$
(f) $P(X < 4)$
(g) $P(X \leq 2)$
(h) $P(X > 2)$
(i) $P(X \geq 2)$
(j) $P(1 \leq X \leq 4)$
(k) $P(1 < X \leq 4)$
(l) $P(1 \leq X < 4)$
(m) $P(1 < X < 4)$
(n) $P(1.5 < X < 4.5)$
(o) $P(3 < X < 4)$

(n) Calculate the expected value, or mean, of $X$.
(o) Calculate the variance of $X$.
(p) Calculate the standard deviation of $X$.
(q) Calculate the skewness of $X$.
(r) Calculate the kurtosis of $X$.

Let $X$ represent the bending strength (in $N/mm^2$) of timber samples for a given species of trees. After performing a large number of tests it is found that the probability density function of $X$ is given by

$$f(x) = \begin{cases} c \cdot \sqrt{x}, & \text{for } 10 < x < 40 \\ 0, & \text{otherwise} \end{cases}$$

(a) Calculate the value of $c$ so that $f(x)$ is a proper probability density function (pdf) for $X$.
(b) Plot the probability mass function (pmf) for $X$.
(c) Obtain an expression for the cumulative distribution function (CDF) of $X$, i.e., $F(x)$.
(d) Plot the cumulative distribution function (CDF) for $X$.

Calculate the following probabilities:

(e) $P(X = 30)$
(f) $P(X \leq 25)$
(g) $P(X < 25)$
(h) $P(X > 25)$
(i) $P(X \geq 25)$
(j) $P(15 \leq X \leq 35)$
(k) $P(15 < X \leq 35)$
(l) $P(15 \leq X < 35)$
(m) $P(15 < X < 35)$

(n) Calculate the expected value, or mean, of $X$.
(o) Calculate the variance of $X$.
(p) Calculate the standard deviation of $X$.
(q) Calculate the skewness of $X$.
(r) Calculate the kurtosis of $X$.
(s) Let $Y = g(X) = \sqrt{X}$ represent the maximum elongation, in $mm$, of the timber piece. What is the expected value of the elongation $Y$?
(t) What is the variance of $Y$?
Let $X$ represent the time to failure, in months, of a delicate electronic component in a probe. Data from the manufacturer suggests that the probability density function for $X$, $f(x)$, can be represented by the triangular diagram shown below. [Note: this pdf represents a triangular distribution with the ordinate $c$ known as the mode of the distribution].

(a) Calculate the value of $c$ so that $f(x)$ is a proper probability density function (pdf) for $X$, if $a = 30$ and $b = 10$.

(b) Write out the expressions for the probability density function $f(x)$. This will be a piecewise function of the form

$$f(x) = \begin{cases} f_1(x), & \text{for } 0 < x < a \\ f_2(x), & \text{for } a < x < a + b \\ 0, & \text{otherwise} \end{cases}$$

(c) Obtain an expression for the cumulative distribution function (CDF) of $X$, i.e., $F(x)$.

(d) Plot the cumulative distribution function (CDF) for $X$.

Calculate the following probabilities:

(e) $P(X = 30)$   (e) $P(X \leq 25)$   (f) $P(X < 25)$   (g) $P(X \leq 15)$

(h) $P(X > 15)$   (i) $P(X \geq 15)$   (j) $P(15 \leq X \leq 35)$   (k) $P(15 < X \leq 35)$

(l) $P(15 \leq X < 35)$   (m) $P(15 < X < 35)$

(n) Calculate the expected value, or mean, of $X$.

(o) Calculate the variance of $X$.

(p) Calculate the standard deviation of $X$.

(q) Calculate the skewness of $X$.

(r) Calculate the kurtosis of $X$.

Two rivers, one carrying a discharge $X$ and another carrying a discharge $Y$, join together into a third river with a discharge $Z$. Because of seepage losses in the tributary rivers the discharge $Z$ is given by $Z = 0.9X + 0.85Y$. (a) If the expected values of discharges $X$ and $Y$ are $E(X) = 135$ cfs (cubic feet per second) and $E(Y) = 85$ cfs, what is the expected value of the discharge $Z$? (b) If the variances of the discharges $X$ and $Y$ are $\text{Var}(X) = 100$ cfs$^2$ and $\text{Var}(Y) = 25$ cfs$^2$, what is the variance of the discharge $Z$? (c) What is the standard deviation of the discharge $Z$?