$\underline{\underline{A}}$ = square matrix, e.g., $\underline{\underline{A}} = \begin{bmatrix} 2 & 4 \\ 4 & -2 \end{bmatrix}$

Eigenvalue problem: $\underline{\underline{A}}\,\underline{x} = \lambda \underline{x} \Rightarrow$ λ = eigenvalues, $\underline{x}$ = eigenvectors

equivalent to: $(\underline{\underline{A}} - \lambda \underline{\underline{I}})\underline{x} = \underline{0}$

$\underline{\underline{I}}$: identity matrix

For matrix $\underline{\underline{A}}_{2\times2} \Rightarrow \underline{\underline{A}}\,\underline{x} = \lambda \underline{x} \Rightarrow \begin{bmatrix} 2 & 4 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\left.\begin{array}{l} 2x_1 + 4x_2 = \lambda x_1 \\ 4x_1 - 2x_2 = \lambda x_2 \end{array}\right\}$$

$$\Leftarrow \left.\begin{array}{l} (2-\lambda)x_1 + 4x_2 = 0 \\ 4x_1 + (-2-\lambda)x_2 = 0 \end{array}\right\}$$

$$\begin{bmatrix} 2-\lambda & 4 \\ 4 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \leftarrow (\underline{\underline{A}} - \lambda \underline{\underline{I}})\underline{x} = 0$$

also,

$$\underline{\underline{\Delta}} = \underline{\underline{A}} - \lambda \underline{\underline{I}} = \begin{bmatrix} 2 & 4 \\ 4 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2-\lambda & 4 \\ 4 & -2-\lambda \end{bmatrix}$$

characteristic matrix

Same

$$(\underline{\underline{A}} - \lambda \underline{\underline{I}})\underline{x} = \begin{bmatrix} 2-\lambda & 4 \\ 4 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (2-\lambda)x_1 + 4x_2 \\ 4x_1 + (-2-\lambda)x_2 \end{bmatrix}$$

In order for $(\underline{\underline{A}} - \lambda \underline{\underline{I}})\underline{x} = \underline{0}$ to have a non-trivial solution (NOTE: Trivial solution $\underline{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$), $\underline{\underline{\Delta}}$ must be singular, i.e., $\det(\underline{\underline{\Delta}}) = 0$

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = \det \begin{bmatrix} 2-\lambda & 4 \\ 4 & -2-\lambda \end{bmatrix} = -(2-\lambda)(2+\lambda) - 16 = -(4-\lambda^2) - 16$$

$$= \lambda^2 - 20 = 0 \leftarrow \text{characteristic equation}$$

$$p(\lambda) = \det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = \lambda^2 - 20 \leftarrow \text{characteristic polynomial}$$

eigenvalues are the zeros of char. polynomial

$\lambda = \pm\sqrt{20}$

In Matlab:

```
>> c = poly(A)
>> p = poly2sym(c, 'r')
```