Background material for the Scilab script "CompHydraulicsII_Script.sce" based on Chapter 4 - from Vreugdenhil, C.B., 1989, "Computational Hydraulics - An Introduction," Springer-Verlag, Berlin.

Material from Chapter 4 - Transport of a Dissolved Substance

Consider the flow in a river of cross-section *A*, if *Q* represents the water discharge, and *T* is a relaxation time for decay of a contaminant, the governing equation for the contaminant concentration *c* is given by

$$\frac{\partial}{\partial t}(Ac) + \frac{\partial}{\partial x}(Qc) + \frac{A}{T}c = 0$$

The equation of water continuity is given by

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \, .$$

Combining these two equations result in the equation

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + \frac{c}{T} = 0 \, .$$

If no decay is present (i.e., $T \rightarrow$ ¥), then the equation above can be written as $dc/dt = 0$ along a trajectory (characteristic) $dx/dt = u$.

A numerical solution to this equation can be attempted by using the finite difference approximations

$$\left.\frac{\partial c}{\partial t}\right|_j^n \approx \frac{c_j^{n+1} - c_j^n}{\Delta t} \text{, and } \left.\frac{\partial c}{\partial x}\right|_j^n \approx \frac{c_{j+1}^n - c_{j-1}^n}{2\Delta x}$$

where $c_j^{n+1} = c(x_j, t_n)$, $j = 0,1,2,\ldots,J$ and $n = 0,1,2,\ldots$. With these approximations the governing equation becomes

$$\frac{c_j^{n+1} - c_j^n}{\Delta t} + u\frac{c_{j+1}^n - c_j^n}{2\Delta x} = 0 \, ,$$

from which an *explicit* equation for solving for c_j^{n+1} is obtained:

$$c_j^{n+1} = c_j^n - \frac{u\Delta t}{2\Delta x}(c_{j=1}^n - c_{j=1}^n) \, .$$

Boundary conditions: At $j = 0$ (left boundary), typically $c_0^n = c_0(t_n)$ is given. At $j = J$ (right boundary), the equation to use is:

$$\frac{c_J^{n+1} - c_J^n}{\Delta t} + u\frac{c_J^n - c_{J-1}^n}{\Delta x} = 0 \, .$$

~~A function called *conc.sci* was developed to calculate the explicit scheme shown above. Here is a listing of the function:~~