

Background material for the Scilab script "CompHydraulicsVI_Script.sce" based on Chapter 7 - from Vreugdenhil, C.B., 1989, "Computational Hydraulics - An Introduction," Springer-Verlag, Berlin.

Material from Chapter 7 - Groundwater flow in a horizontal layer

The equation to be solved is

$$\frac{\partial h}{\partial t} = D \frac{\partial^2 h}{\partial x^2} + \frac{w}{n}$$

where h is the groundwater surface elevation above a horizontal impermeable layer, x is distance along the horizontal direction, t is time, D is a diffusion coefficient (m^2/s), w is the rainfall in volume per unit surface area (i.e., depth), and n is the soil porosity.

The example shown in Vreughdenhil's book corresponds to an aquifer next to a river so that the water level in the river is lower by an amount Δh from the aquifer's water level h_o . The exact solution is given by

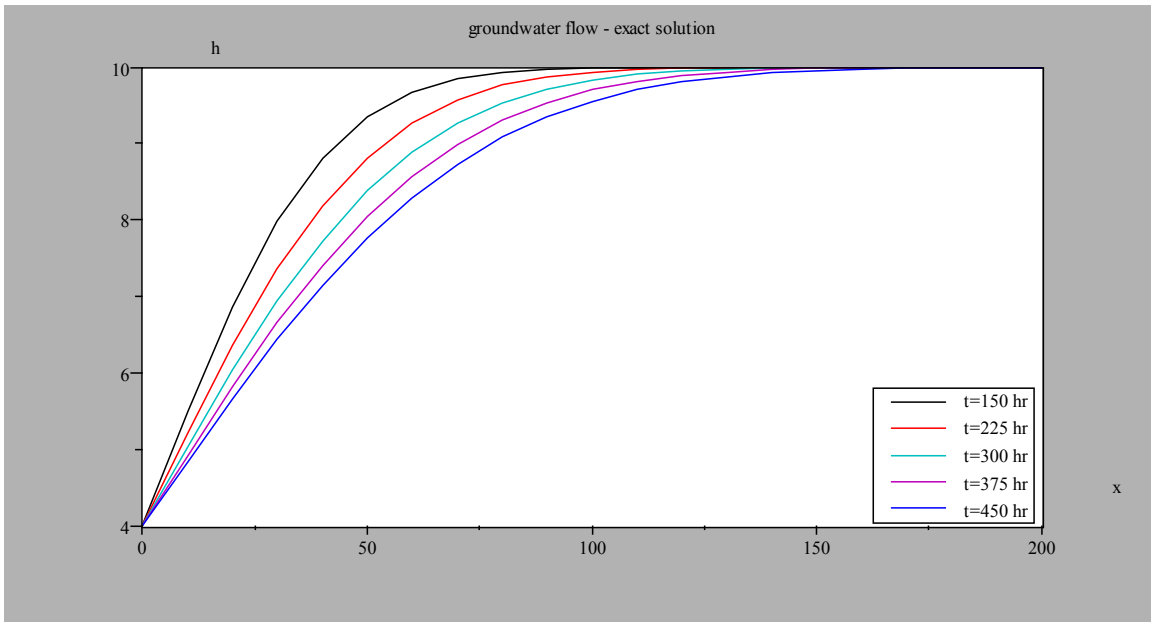
$$h(x, t) = h_o - \Delta h \cdot \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Where erfc is the complementary error function, i.e., $\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt$.

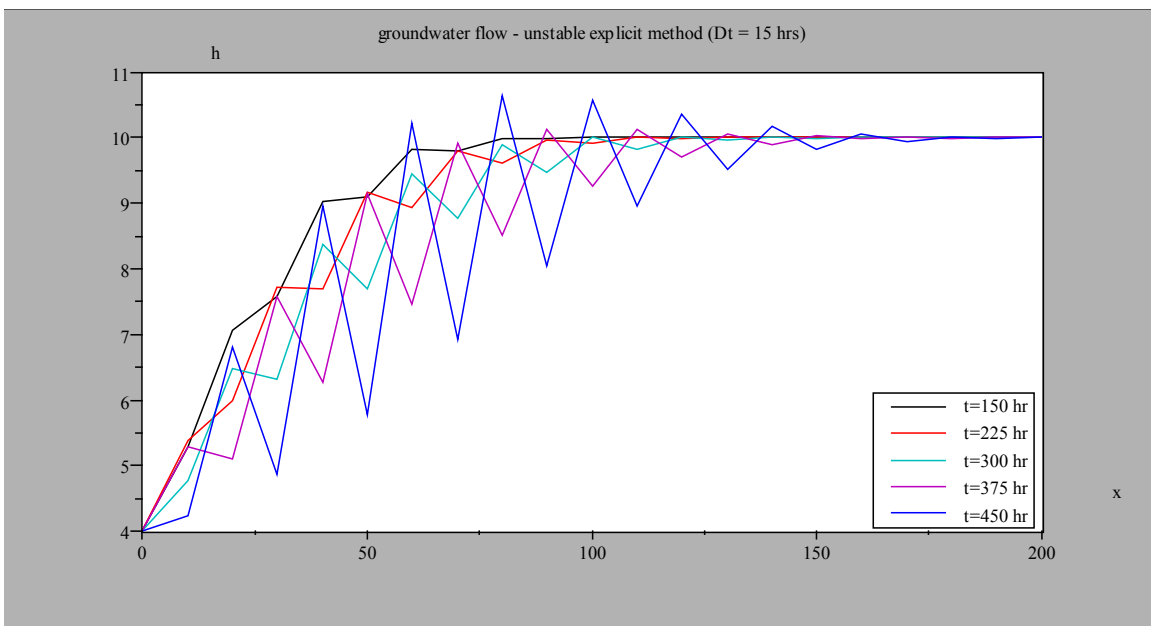
The numerical solution is achieved by using the forward-in-time, centered-in-space (FTCS) explicit scheme, given in equation (7.7) in Vreugdenhill's book, with $D = 0.001 m^2/s$, $\Delta x = 10 m$, and $\Delta t = 15 hrs$. This solution produces instability due to the relatively large time step (The scheme is stable if the diffusion parameter, $\lambda = 2 \cdot D \cdot \Delta t / \Delta x^2 \leq 1$). The solution is repeated for a time step $\Delta t = 7.5 hrs$, which produces a stable explicit solution. Function `gwexplicit.m`, calculates the explicit solution.

Additional numerical solutions are provided by using an implicit scheme (equation 7-12, page 39, in Vreugdenhill's book) which produces a tri-diagonal matrix of coefficients. This matricial equation is solved by using Thomas' algorithm (pages 40 and 41 in Vreughdenhill's book). Function `thomas.m` implements the implicit solution. The solutions are driven by the script `CompHydExVI_Script.m`.

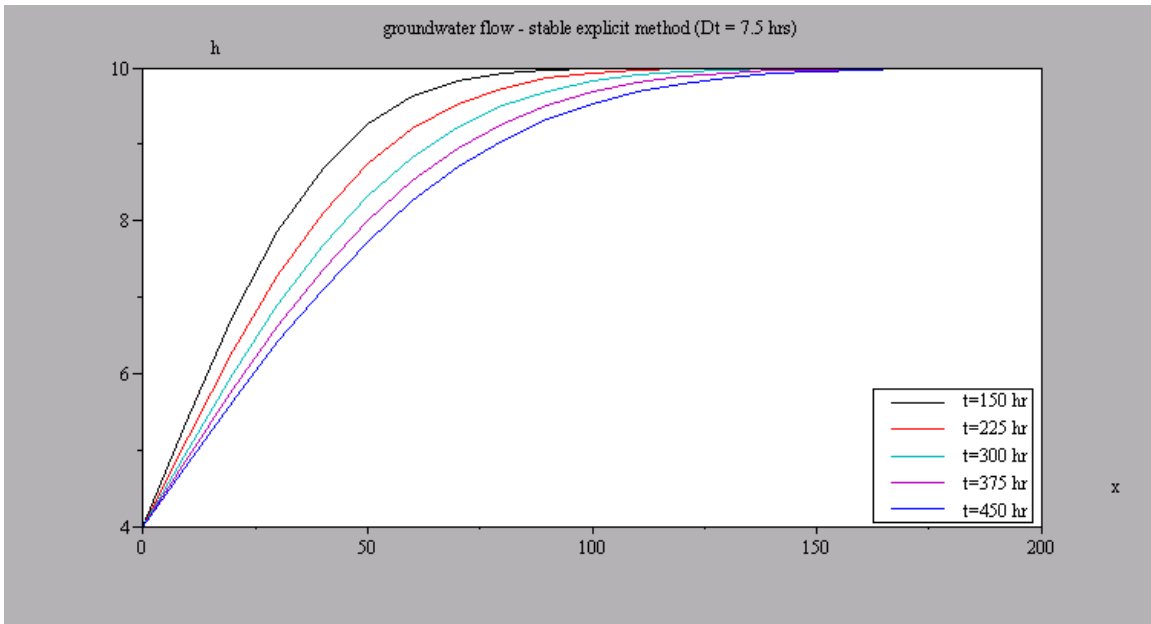
The following figures show the results of the exact and numerical solutions with different solution schemes and solution parameters. First, we show the exact solution showing the water depth as a function of time:



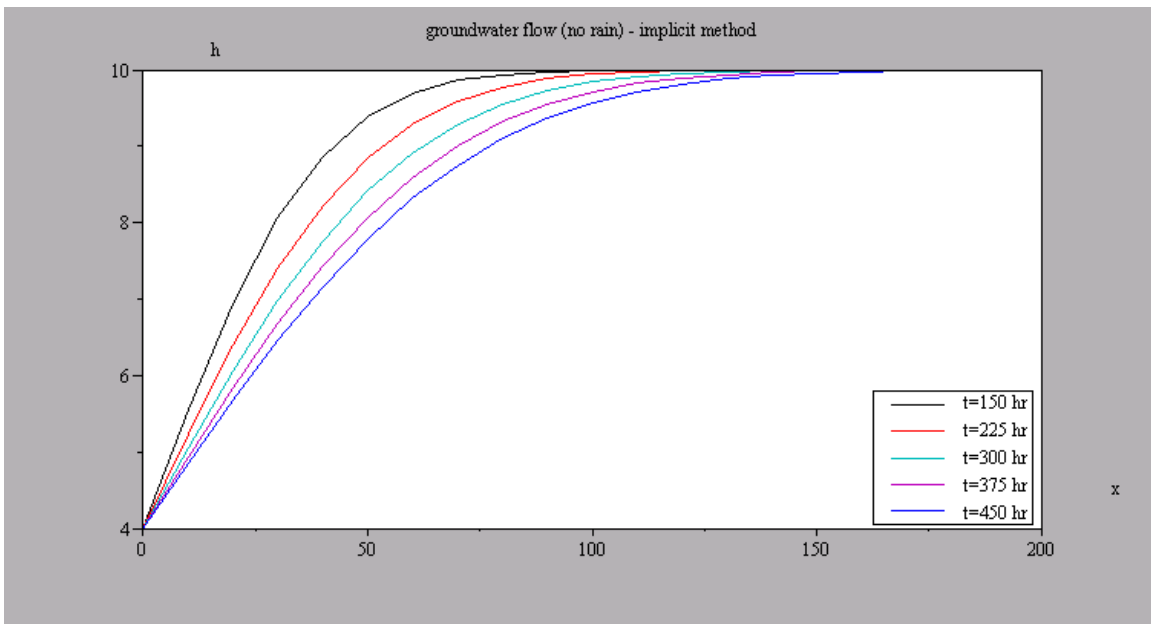
Next, we show the unstable explicit method corresponding to $\Delta t = 15$ hrs:



The following figure shows the stable explicit method corresponding to $\Delta t = 7.5$ hrs:



The following figure shows the solution using an implicit method (no rain):



The following figure shows the implicit method solution including rain:

