

Background material for the Matlab script "CompHydExV_Script.m" based on Chapter 6 - from Vreugdenhil, C.B., 1989, "Computational Hydraulics - An Introduction," Springer-Verlag, Berlin.

Material from Chapter 6 - Kinematic Waves

The equation to be solved is

$$\frac{\partial y}{\partial t} + c \frac{\partial y}{\partial x} = 0, \tag{I}$$

where y is the water depth in a river (notice that Vreugdenhil's book uses a instead of y), x is the longitudinal coordinate along the river path, and t is time. The velocity of propagation of the kinematic wave is given by

$$c = \frac{1}{B} \sqrt{\frac{g \cdot i}{c_f}} \frac{d}{dy} (A_s \sqrt{R}),$$

where B is the surface (or storage) width, R is the hydraulic radius, A_s is the cross-sectional area, g is gravity, i = river bed slope, c_f is bed friction coefficient defined by

$$c_f \cdot \frac{Q |Q|}{R \cdot A_s^2} = g \cdot i,$$

and Q is the water discharge.

Notice that the velocity of propagation of the kinematic wave, c , is not a constant since it depends on the depth y . For a wide, rectangular cross-section, $A_s = B_s y$, and the velocity of propagation c reduces to

$$c = \frac{3}{2} u \frac{B_s}{B},$$

which is of the same order of magnitude as the flow velocity, $u = Q/A_s$.

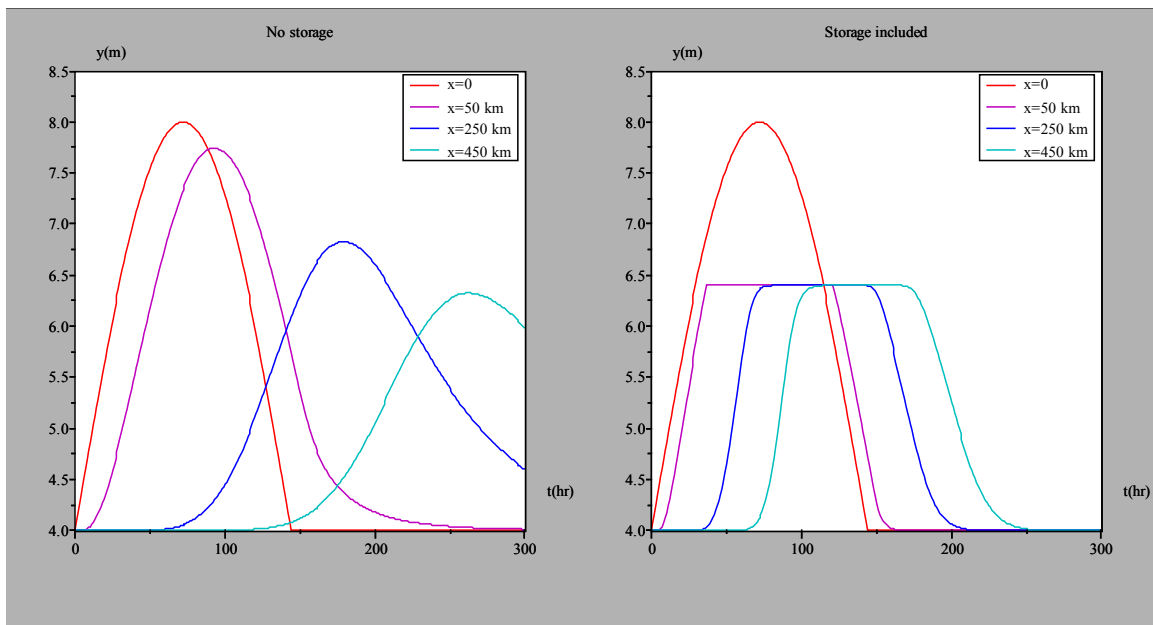
Notice that equation (I) is the convection equation and can be solved using the *modified Lax method* first presented in Vreughdenhill's Chapter 5, equation (5.1).

Examples of kinematic wave calculation

The following examples calculate the water depth for a kinematic wave with the parameters $B_s = B = 200 \text{ m}$, $c_f = 0.004$, $i = 0.0001$. Initially, a flow $Q = 800 \text{ m}^3/\text{s}$ at a depth of $y = 4 \text{ m}$ is assumed to flow through the river. At the upstream end of the channel a single sinusoidal depth wave with a maximum height of 4 m above the equilibrium depth and having a period $T = 6 \text{ days}$ is assumed to enter the channel. This boundary is located at a distance of 450 km upstream of the end of the river.

In the first example, the kinematic wave travels through the river without damping, while in the second example a storage basin with a surface area of $5 \times 10^8 \text{ m}^2$ is provided for. The basin will start filling when the local water level exceeds 6.40 m .

Function *kinematicwave.sci* calculates the kinematic wave travel without damping, while function *kinematicwave2.sci* incorporates damping. The script *CompHydExV_Script.m* drives the calculations. The figure below shows some results.



The figure shows the water depth, $y(m)$, as a function of time, $t(hr)$, at different locations in the river. The location $x = 0$ corresponds to the upper end of the river, while the location $x = 450$ km corresponds to the end of the river. Notice that the wave moves down the river and suffers dissipation as it travels. The second figure shows the wave damping due to the presence of the storage basin. The effect of the basin is to "chop" the peak off the traveling kinematic wave.

This document was prepared by Gilberto E. Urroz, Associate Professor, Department of Civil and Environmental Engineering, Utah State University, September 2004. The solutions are based on the book Vreugdenhil, C.B., 1989, "Computational Hydraulics - An Introduction," Springer-Verlag, Berlin.