

Background material for the Matlab script "CompHydExVI\_Script.sce" based on Chapter 11 - from Vreugdenhil, C.B., 1989, "Computational Hydraulics - An Introduction," Springer-Verlag, Berlin.

Material from Chapter 11 - Convection-Diffusion (Transport of a Dissolved Substance)

The equation to be solved is

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2}$$

where  $c$  is the concentration of a substance being transported in a one-dimensional flow in the  $x$  direction with a velocity  $u$ . The substance is also affected by diffusion, represented by the term in the right-hand side of the equation above. The parameter  $D$  is known as the diffusion coefficient.

The numerical solution is achieved by using the Crank-Nicholson method shown in Vreugdenhil's book, page 58:

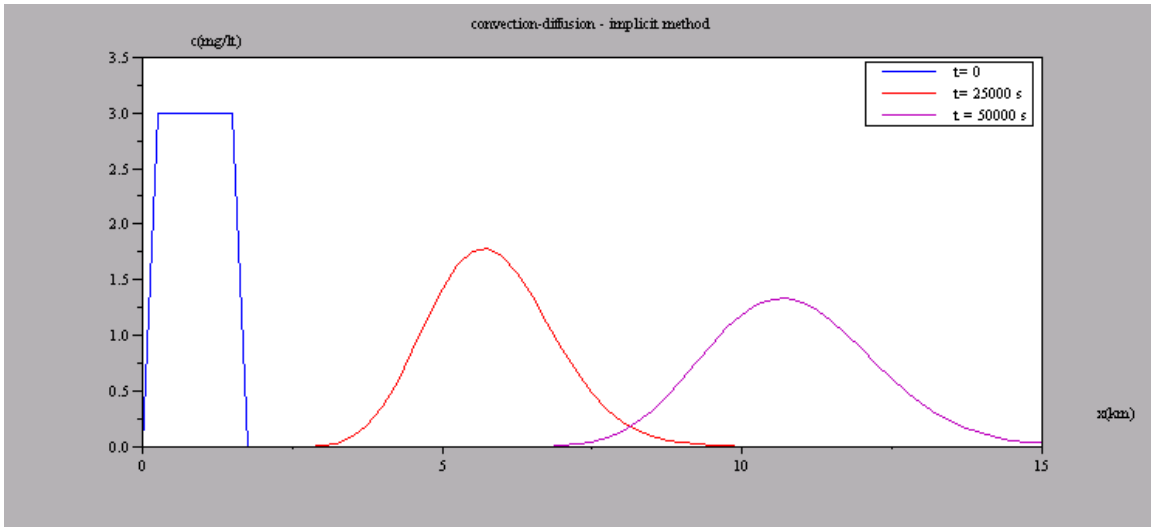
$$\frac{c_j^{n+1} - c_j^n}{\Delta t} + \theta \left\{ \frac{c_{j+1}^{n+1} - c_{j-1}^{n+1}}{2\Delta x} - D \frac{c_{j+1}^{n+1} - 2c_j^{n+1} - c_{j-1}^{n+1}}{\Delta x^2} \right\} + (1 - \theta) \left\{ \frac{c_{j+1}^n - c_{j-1}^n}{2\Delta x} - D \frac{c_{j+1}^n - 2c_j^n - c_{j-1}^n}{\Delta x^2} \right\} = 0$$

This equation can be re-written in terms of the parameters  $\lambda = 2D\Delta t/\Delta x^2$ , and  $\sigma = u\Delta t/\Delta x$ . The implementation is shown in page 59 of the book.

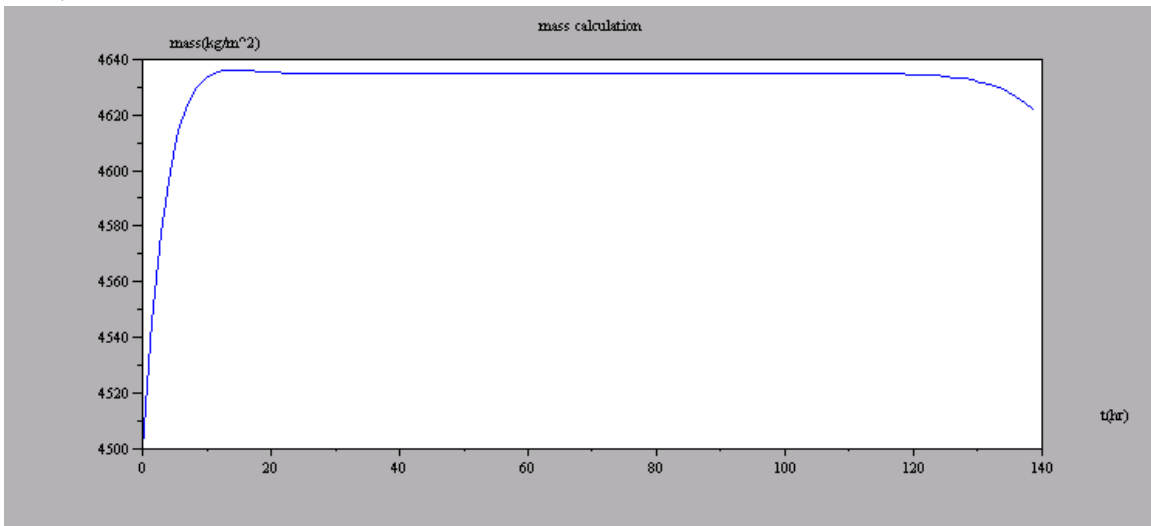
Function *convectiondiffusion.m* implements the numerical solution.

The example solved in the book corresponds to a "cloud" of contaminant suddenly dumped in a 1500-m long interval at the upstream end of a river flowing with constant velocity  $u = 0.02$  m/s. The problem is solved on a 15 km reach of the river. The diffusion coefficient is  $D = 1.6$  m<sup>2</sup>/s. We use the Crank-Nicholson method for the solution with  $\theta = 0.55$ ,  $\Delta x = 250$  m, and  $\Delta t = 5000$  s. We perform a second solution corresponding to a continuous injection of contaminant for the period  $0 < t < 50000$ , and a third one for continuous injection throughout the simulation time. We also calculate the mass of contaminant against time for all cases. The script *CompHydExVII\_Script.m* drives the solution.

Some of the results are shown next. First, we show the contaminant concentration distribution at different times:

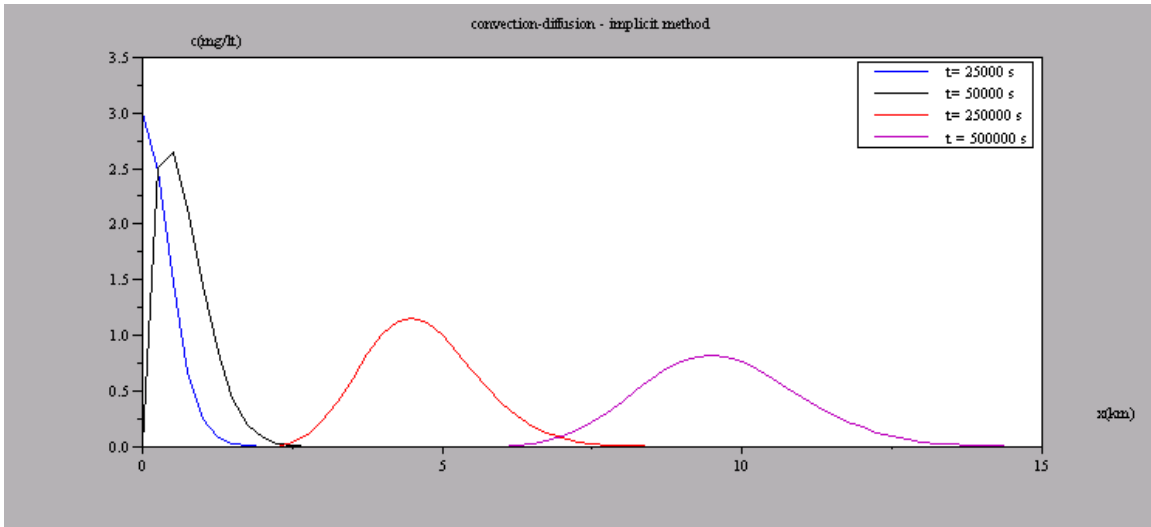


Next, we show the mass calculation with time:

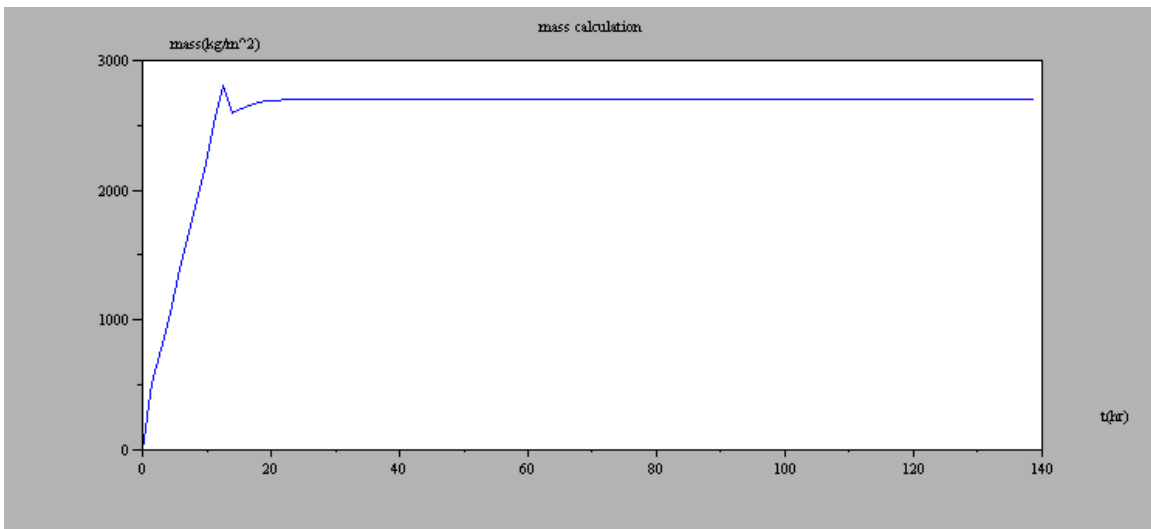


Notice that the mass remains constant through the entire interval of time, except near the start and end of the simulation. The increase in mass at the start of the simulation reflects the injection of contaminant into the river. The discrepancy at the end of the simulation is due to the numerical approximation involved in the prediction of the contaminant concentration.

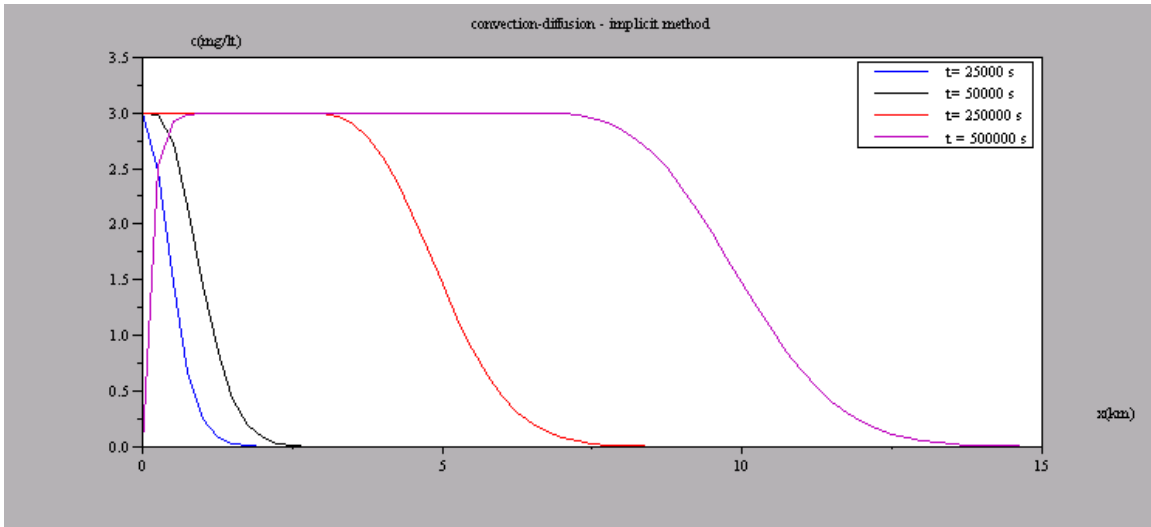
The next figure shows the concentration distribution for different times for the case of a continuous injection for the period  $0 < t < 50000$ :



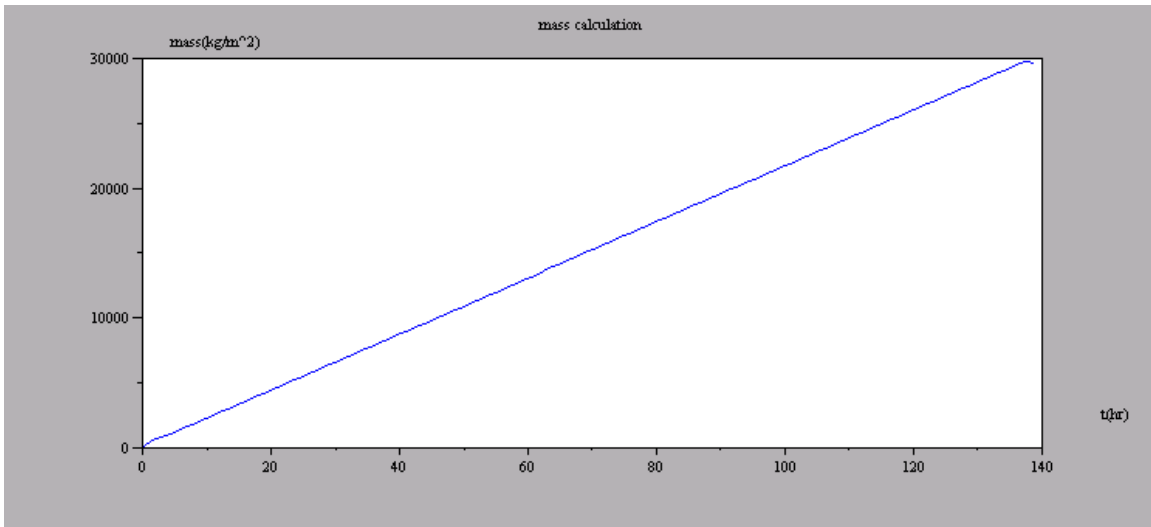
The corresponding mass calculation is shown next:



The following figure shows the contaminant concentration for a continuous injection of contaminant:



The mass calculation for this case is shown next:



In this case, the mass of contaminant increases linearly with time since the continuous injection of contaminant at the upstream end of the river ensures the increase of the contaminant mass with time.