CEE 6510 – Fall 2004 – Assignment No. 7

[1] Kinematic wave solution using the leap-frog scheme.
Program the kinematic wave problem of Chapter 6 using the leap-frog method (Section 5.2, pp. 25-26 in Vreugdenhill's book). Download the file *CompHydV_Scripts.zip* to see the solution using the Lax method (section 5.1, pp. 23-25). You can modify the files in *CompHydV_Scripts.zip* to produce your leap-frog program. Solve the two problems indicated in Chapter 6, i.e., (a) the problem of the kinematic wave without a retention basin, and (b) the problem including a retention basin. Let your program produce graphs as those shown in page 33 of Vreugdenhil's book.

[2] Flow in groundwater layer with tidal boundary condition.
Refer to the problem illustrated in Figure 7.2, p. 35, in Vreugdenhill's book. This problem postulates a sudden drop in the water surface in a river adjacent to a groundwater layer which extends to infinity in the opposite direction. The solution to this problem is provided in the file *CompHydVI_Scripts.zip*. Suppose that instead of being adjacent to a river, the groundwater layer is adjacent to an ocean whose surface is subject to a tidal fluctuation described by the following boundary condition at $x = 0$:

$$h_u(t) = h(0,t) = h_0 + \Delta h \cdot \sin\left(\frac{2\pi t}{T}\right),$$

where t is time, h_o is the equilibrium water level in the groundwater layer, and T is the tidal period = $24\ hrs$. At $x = L$, the boundary condition is $h(L,t) = h_0$. The "diffusion" coefficient for the groundwater layer problem is given by $D = ka$, where k is the permeability (or hydraulic conductivity) of the layer, and a is the layer thickness (or water depth above an impervious layer). For this solution assume that $D = 10^{-3}\ m^2/s$. Use a length of $L = 60\ m$, a length increment of $\Delta x = 2\ m$, a maximum time of $t_{mzx} = 24\ hrs$, and a time increment of $\Delta t = 0.5\ hrs$ to simulate the aquifer response to the tidal wave. (a) Modify the function *thomas.m* to solve the problem described above. Produce a movie of the oscillating water table in the aquifer. Produce plots of the water depth as function of time at points $x = 0, L/4, L/2, 3L/4$. (b) If you were to use the boundary condition $\partial h/\partial x = 0$ at $x = L$, what is the maximum value of L that you can use such that the aquifer depth fluctuations at that point are less than $0.5\ ft$? [NOTE: the case described in (b) corresponds to an aquifer subject to the same oscillating boundary conditions at $x = 0$ and $x = 2L$, so that, by considerations of symmetry, we can use the boundary condition: $\partial h/\partial x = 0$ at $x = L$. The solution for part (b) will require you to try values of L from 20 ft to 200 ft and quantify the amplitude of the solution at $x = L$ until you find the value of L for which the amplitude does not grow larger than $0.5\ ft$.]

[3] Unsteady, plane Poiseuille flow.
Solve the problem of developing flow in a plane Poiseuille flow as described in the handout *PoiseulleFlow.pdf*.

[4] Dispersion of a Gaussian concentration distribution in a moving flow.
Chapter 11 in Vreugdenhill's book describes the problem of convection-diffusion (dispersion) of a contaminant cloud in a one-dimensional moving fluid. The problem described in that Chapter uses a constant concentration distribution as initial conditions and shows its dispersion with time for a fluid moving with a constant velocity. This solution is available in the file *CompHydVII_Scripts.zip*. The Matlab files also provide the solution for a constant concentration injection lasting a finite period of time, as well as for a permanent constant concentration injection. For the problem proposed herein, use $D = 1.6\ m^2/s$, $L = 15\ km$, $\Delta x = 250\ m$, $t_{max} = 6\ days$, $\Delta t = 5000\ s$, $\theta = 0.55$, and $u = 0.02\ m/s$. Assume zero concentration in the flow at $t = 0$ (initial conditions), and use the following boundary condition at $x = 0$, with $\sigma_x = 2000\ m$.

$$c_b(t) = 3.0 \cdot \exp\left(-\frac{u^2 t^2}{2\sigma_x^2}\right),$$

where u is the constant flow velocity. (a) Solve the problem described above by using function *convectiondiffusion.m*. (b) Show a movie of the concentration distribution on the domain $0 < x < L$ for different times. (c) produce a graph of concentration distributions at times $t = 0$, $t = 2.5\ days$, and $t = 5.0\ days$. (d) Using function *inttrap* calculate the mass of contaminant in the domain as a function of time, and produce a plot of mass vs. time.

Submit:

- Word document showing problem statement, functions or scripts used to solve problem, and output showing solution. Include your name in this document.
- M-files with Matlab scripts and functions.
- Input or output text files.

How to submit your assignment:

Place the Word document, *m*-files, and input and output text files in a zip file. E-mail the zip file to: gurro@cc.usu.edu