CEE 6510 – Fall 2004 – Assignment No. 5

[1]. **Programming numerical solution for ODE** – (a) Write a Matlab function to solve the explicit and implicit forms of a second-order ordinary differential equation for the example outlined in the document *Numerical solution to ordinary differential equations*. Let one of the arguments to the function be a string that takes as value either 'e' for *explicit*, or 'i' for *implicit*, so that the user can decide which method to use. (b) Solve the differential equation $d^2y/dx^2 = -y$, with initial conditions $y(0) = 1$, $y'(0) = -1$ numerically by using the function developed in (a). Solve it using both the explicit and implicit methods in the interval $0 \leq x \leq 2$, for values of the increment $\Delta x = 0.20, 0.10, 0.05, 0.01$. Plot the results of the explicit method and of the implicit method separately identifying the curves corresponding to each value of Δx in the graphs. (c) What effect, if any, does the increment Δx has on the numerical solution?

[2]. **Water quality in a lake – part 2** – Referring to the case presented in Chapter 2 of Vreughdenhil's book (Computational Hydraulics), the governing differential equation is:

$$\frac{dc}{dt} + \frac{c}{T} = \frac{c_e}{T},$$

where c = lake BOD concentration, T = system time scale = $V/(Q+V/T_r)$, V = lake volume, Q = lake inflow or outflow (constant), T_r = time scale for degradation, and c_e = concentration excitation function. Assume that the function c_e is given by a decaying oscillating function, namely:

$$c_e(t) = \exp\left(-\frac{t}{T_o}\right) \cdot \left(ce_0 + ce_1 \cdot \cos\left(\frac{2\pi \cdot t}{T_e}\right)\right),$$

where T_o, T_e are time scales for decay and oscillation, respectively, and ce_0 and ce_1 are constants. (a) Using Matlab function *ode23*, obtain numerical solutions to the governing differential equation if the initial condition is $c(0) = c_o$. Use the values $Q = 1\ m^3/s$, $V = 6x10^5\ m^3$, $T_r = 3$ *days*, $T_0 = 2$ *days*, $T_e = 5$ *days*, $c_i = 5$ *mg/lt*, $ce_0 = 20$ *mg/lt*, and $ce_1 = 5$ *mg/lt*, and plot both $c_e(t)$ - *vs*- t and the numerical solution for the lake concentration found, $c(t)$ - *vs* - t, in the interval $0 < t < 10$ *days*, for $c_0 = 2.5$ *mg/lt*. (b) Repeat part (a) using Matlab function *ode45* and the same initial condition. Plot the results for $c(t)$ - *vs* - t from (a) and (b) in the same graph. (c) Is there a significant difference in the solutions provided by functions *ode23* and *ode45* for this case? (d) Repeat the solution in (b) using Matlab function *ode45* but with initial conditions of $c_0 = 0$, *1.25 mg/lt*, and *2.5 mg/lt*. (e) How does the value of the initial condition affect the numerical solution?

[3]. **Method of characteristics for hydraulic transients** – Solve the exercise in page 11 of the document *TransientCharacteristics.pdf*.

Submit:

- Word document showing problem statement, functions or scripts used to solve problem, and output showing solution. Include your name in this document.
- M-files with Matlab scripts and functions.
- Input or output text files.

How to submit your assignment:

Place the Word document, *m*-files, and input and output text files in a zip file. E-mail the zip file to: gurro@cc.usu.edu