CEE 6510 – Fall 2004 – Assignment No. 4

[1]. **Data fitting problem - Infiltration data**. Infiltration into soil is typically modeled using Horton's equation

$$f = f_c + (f_0 - f_c) \cdot e^{-kt},$$

where f is the infiltration rate, f_c is the infiltration rate at saturation, f_0 is the initial infiltration rate, t is time, and k is a constant that depends primarily on the type of soil and vegetation of the area of interest.

The following table shows measurements of the infiltration rate, f, as function of time, t, for a specific storm in a watershed.

t(hr)	*f(cm/hr)*	*t(hr)*	*f(cm/hr)*
1	3.9	14	1.43
2	3.4	16	1.36
3	3.1	18	1.31
4	2.7	20	1.28
5	2.5	22	1.25
6	2.3	24	1.23
8	2	26	1.22
10	1.8	28	1.2
12	1.54	30	1.2

(a) Use the least-square approach described in Problem [4], Assignment No. 2, to determine the values of the parameters f_c, f_0, and k for Horton's equation using the data in the previous table. Before using that function, however, it is necessary to linearize the relationship, $f = f_c + (f_0 - f_c)e^{-kt}$, by writing it as $\eta = f - f_c = (f_0 - f_c)e^{-kt}$, or $\eta = ae^{-kt}$, with $a = f_0 - f_c$. This relationship can then be written as $\ln \eta = \ln a - kt$, and a linear regression fitting can be attempted.

However, f_c, which is necessary to calculate η, is not known *a priori*. We do know, however, that f_c is the infiltration rate at saturation, i.e., the value of f as $t \to \infty$. Thus, the value of f for large values of t (see table), namely, $f_c = 1.2$, can be used as a first guess for f_c. Then, values of η can be calculated and the least-square method used to produce the coefficients a and k.

Afterwards, we can proceed to plot the original and fitted data. If the graph shows too large a discrepancy between original and fitted data, we can try different values of f_c smaller than 1.2, say, $f_c = 0.8, 0.9, 1.0, 1.1,$ *etc.*, until we find one that produces good graphical fitting.

(b) Plot the original data as points together with the best fitted equation as a line in the same graph.

(c) The volume of water infiltrated per unit area (i.e., the depth of infiltration) is calculated as $h = \int_0^t f(t)dt$. Using a numerical integration of the Horton's equation with functions *quad* and *quad8* determine the depth of infiltration after 1, 2, and 5 hours, assuming that there is a constant water supply at the surface.

(d) Plot the depth of infiltration h vs. time for $0 < t < 10$ *hours*.

[2] **Interpolation problem - Movable bed profile.** The following data shows the elevation of the bed surface in a movable-bed flume, *y(ft)*, against the distance from one of the flume walls, *x(ft)*.

x(ft)	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*	*3.5*	*4.0*
y(ft)	*0.23*	*0.45*	*0.48*	*0.40*	*0.31*	*0.40*	*0.59*	*0.37*

(a) Using polynomial interpolation of order 2, 3, 4, 5, and 6 for the bed profile data, plot the original data as points and the fitted polynomials as a continuous lines of different colors.

(b) Use cubic splines to interpolate the bed profile data. Plot the original data as points and the fitted splines as continuous lines of different colors.

[3]. **Multiple-linear fitting problem - Precipitation and runoff.** The following table shows data indicating the monthly precipitation (*mm*) during the month of February, P_f, and during the month of March, P_m, as well as the runoff (*mm*) during the month of March, R_m, for a specific watershed during the period 1935-1958.

Year	*Pm*	*Pf*	*Rm*
1935	*9.74*	*4.11*	*6.15*
1936	*6.01*	*3.33*	*4.93*
1937	*1.30*	*5.08*	*1.42*
1938	*4.80*	*2.41*	*3.60*
1939	*4.15*	*9.64*	*3.54*
1940	*5.94*	*4.04*	*2.26*
1941	*2.99*	*0.73*	*0.81*
1942	*5.11*	*3.41*	*2.68*
1943	*7.06*	*3.89*	*4.68*
1944	*6.38*	*8.68*	*5.18*
1945	*1.92*	*6.83*	*2.91*
1946	*2.82*	*5.21*	*2.84*
1947	*2.51*	*1.78*	*2.02*
1948	*5.07*	*8.39*	*3.27*
1949	*4.63*	*3.25*	*3.05*
1950	*4.24*	*5.62*	*2.59*
1951	*6.38*	*8.56*	*4.66*
1952	*7.01*	*1.96*	*5.40*
1953	*4.15*	*5.57*	*2.60*
1954	*4.91*	*2.48*	*2.52*
1955	*8.18*	*5.72*	*6.09*
1956	*5.85*	*10.19*	*4.58*
1957	*2.14*	*5.66*	*2.02*
1958	*3.06*	*3.04*	*2.59*

(a) Use multiple linear fitting to find a relationship of the form $R_m = b_0 + b_1P_m + b_2P_f$.
(b) Plot, in the same graph, the original data for R_m as function of the year as well as the values calculated from the multiple linear fitting.

[4]. **Analytical and numerical solution for ODE** – (a) Use function *dsolve* to solve the differential equation $d^2y/dx^2 = -y$, with initial conditions $y(0) = 1$, $y'(0) = -1$, analytically. (b) Convert the differential equation $d^2y/dx^2 = -y$, into a first-order system of 2 differential equations with $u_1 = y$, and $u_2 = dy/dx$, and solve it numerically using both functions *ode23* and *ode45* with the initial conditions: $y(0) = 1$, $y'(0) = -1$. (d) Plot the analytical solution found in (a) together with the numerical solutions found for functions *ode23* and *ode45*. (e) How close are the numerical solutions to the analytical (exact) solution?

[5]. **Water quality in a lake** - Referring to the case presented in Chapter 2 of Vreughdenhil's book (Computational Hydraulics), the governing differential equation is:

$$\frac{dc}{dt} + \frac{c}{T} = \frac{c_e}{T},$$

where c = lake BOD concentration, T = system time scale = $V/(Q+V/T_r)$, V = lake volume, Q = lake inflow or outflow (constant), T_r = time scale for degradation, and c_e = concentration excitation function. Assume that the function c_e is given by a decaying oscillating function, namely:

$$c_e(t) = \exp\left(-\frac{t}{T_o}\right) \cdot \left(ce_0 + ce_1 \cdot \cos\left(\frac{2\pi \cdot t}{T_e}\right)\right),$$

where T_o, T_e are time scales for decay and oscillation, respectively, and ce_0 and ce_1 are constants. (a) Using functions *dsolve* and *pretty* obtain an analytical solution to the governing differential equation if the initial conditions are $c(0) = c_o$. (b) Using the values $Q = 1\ m^3/s$, $V = 6\times10^5\ m^3$, $T_r = 3$ *days*, $T_0 = 2$ *days*, $T_e = 5$ *days*, $c_0 = 2.5$ *mg/lt*, $ce_0 = 20$ *mg/lt*, and $ce_1 = 5$ *mg/lt*, plot both $c_e(t)$ - *vs*- t and the solution for the lake concentration found in (a), $c(t)$ - *vs* - t, in the interval $0 < t < 10$ *days*.

Submit:

- Word document showing problem statement, functions or scripts used to solve problem, and output showing solution. Include your name in this document.
- M-files with Matlab scripts and functions.
- Input or output text files.

How to submit your assignment:

Place the Word document, *m*-files, and input and output text files in a zip file. E-mail the zip file to: gurro@cc.usu.edu