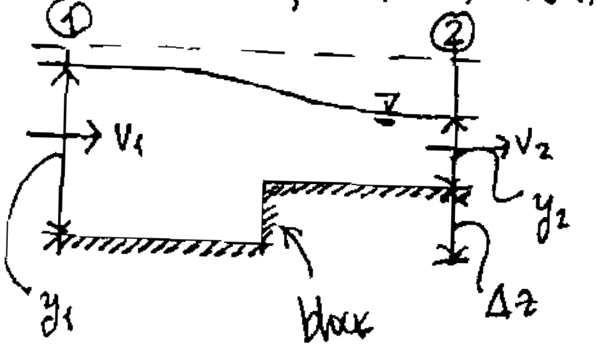
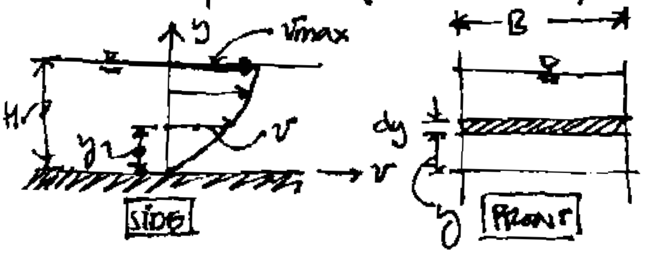
CEE 3500 – TEST 2 SAMPLE 1 – FALL 2005

[1]. Determine the force that the water exerts on the block assuming no energy loss between secions ① and ②. The flow takes place in a rectangular channel of width $B = 2$ ft. Also given: $y_1 = 4.0$ ft, $y_2 = 2.0$ ft, $\Delta z = 1.0$ ft,

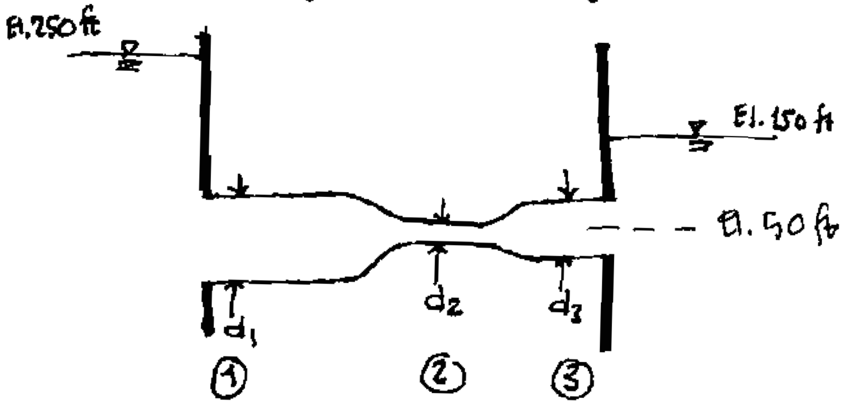


[2]. Laminar flow in an open channel follows a parabolic distribution as shown. Determine the kinetic energy correction factor for this flow. (α).



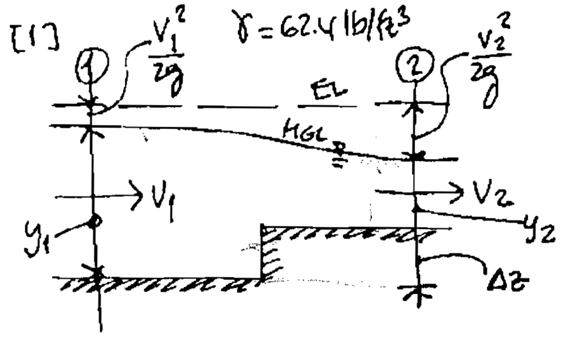
[3]. A reservoir supplies water to a horizontal 6-in pipe 800 ft long. The pipe flows full and discharges into the atmosphere at a rate of 2.23 cfs. What is the pressure in psi midway in the pipe, assuming the only lost head is 6.20 ft in each 100 ft of length?

[4]. Sketch the energy line (EL) and hydraulic grade line (HGL) in the figure below



[5]. For the system of problem [4], determine the pressure (in psi) at secion 2. $d_1 = 1$ ft, $d_2 = 0.25$ ft, $d_3 = 0.50$ ft. Neglect all losses except discharge losses.

[1] $\gamma = 62.4\ \text{lb/ft}^3$



width, $B = 2\text{ft}$

$y_1 = 4.0\text{ ft},\ y_2 = 2.0\text{ ft},\ \Delta z = 1.0\text{ ft}$

∘ continuity, $Q_1 = Q_2 \Rightarrow \cancel{B}y_1V_1 = \cancel{B}y_2V_2$

$$V_1 = \frac{y_2}{y_1}V_2 = \frac{2.0\text{ ft}}{4.0\text{ ft}}V_2 = \frac{V_2}{2} \qquad (A)$$

∘ Energy: $y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z$

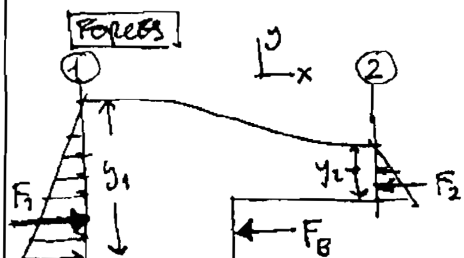
$$\Rightarrow y_1 + \frac{(V_2/2)^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z \Rightarrow y_1 + \frac{V_2^2}{8g} = y_2 + \frac{V_2^2}{2g} + \Delta z \Rightarrow y_1 - y_2 = \frac{V_2^2}{2g} - \frac{V_2^2}{8g} + \Delta z$$

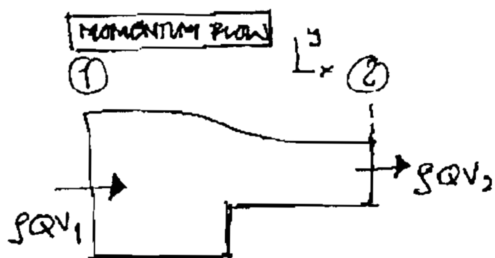
$$\Rightarrow y_1 - y_2 = \frac{V_2^2}{g}\left(\frac{1}{2} - \frac{1}{8}\right) + \Delta z \Rightarrow \text{with } \frac{1}{2} - \frac{1}{8} = \frac{4}{8} - \frac{1}{8} = \frac{3}{8} \Rightarrow y_1 - y_2 = \frac{3}{8}\frac{V_2^2}{g} + 1.0 \Rightarrow$$

$$V_2^2 = \frac{8}{3}g(y_1 - y_2 - 1) = \frac{8}{3} \times 32.2\ \frac{\text{ft}}{\text{s}^2} \times (4.0 - 2.0 - 1.0)\text{ ft} = \cancel{171.73}\ 85.86\ \frac{\text{ft}^2}{\text{s}^2} \Rightarrow \boxed{V_2 = \cancel{13.10}\ 9.26\ \text{ft/s}}$$

From (A) $\boxed{V_1 = \frac{V_2}{2} = \frac{\cancel{13.10\text{ ft/s}}\ 9.26}{2} = \cancel{6.55}\ 4.63\text{ ft/s}}$ and $\boxed{Q = By_1V_1 = 2\text{ft} \times 4\text{ft} \times \cancel{6.55}\ 4.63\ \text{ft/s} = \cancel{52.4}\ 37.04\ \frac{\text{ft}^3}{\text{s}}}$

∘ MOMENTUM





NOTE:
$F_1 = \frac{1}{2}\gamma y_1^2 B$
$F_2 = \frac{1}{2}\gamma y_2^2 B$
hydrostatic forces

$$\Sigma F_x = (\rho QV)_x^{out} - (\rho QV)_x^{in} \Rightarrow F_1 - F_B - F_2 = \rho QV_2 - \rho QV_1$$

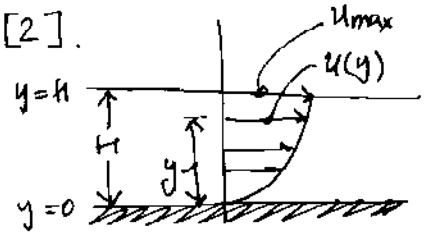
$$\Rightarrow F_B = F_1 - F_2 - \rho QV_2 + \rho QV_1 = \frac{1}{2}\gamma y_1^2 B - \frac{1}{2}\gamma y_2^2 B - \rho Q(V_2 - V_1)$$

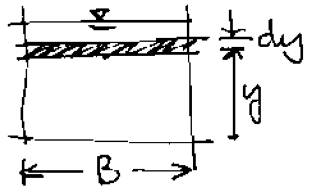
$$F_B = \frac{1}{2}\gamma B(y_1^2 - y_2^2) - \underset{\left[\frac{\gamma}{g}\right.}{\rho} Q(V_2 - V_1) = \underbrace{\frac{1}{2} \times 62.4\frac{\text{lb}}{\text{ft}^3} \times 2\text{ft} \times (4^2 - 2^2)\text{ft}^2}_{748.8\text{ lb}} - \underbrace{\frac{62.4\frac{\text{lb}}{\text{ft}^3}}{32.2\frac{\text{ft}}{\text{s}^2}} \times (\cancel{13.10}\ 9.26 - \cancel{6.55}\ 4.63)\frac{\text{ft}}{\text{s}} \times \cancel{52.4}\ 37.04\ \frac{\text{ft}^3}{\text{s}}}_{332.34}$$

$F_B = \cancel{680.25}\ 416.46\text{ lb} \leftarrow$ FORCE THAT BLOCK EXERTS ON WATER

$\boxed{F_{W/B} = \cancel{680.25}\ 416.46\text{ lb}} \rightarrow$ FORCE THAT ~~BLOCK~~ WATER EXERTS ON BLOCK

[2].





$u(y) = u_{max}\left(\frac{y}{H}\right)^2$

$dA = B\,dy$ $\qquad \alpha = ?$

$$Q = \int_A u\,dA = \int_0^H u_{max}\left(\frac{y}{H}\right)^2 B\,dy = \frac{1}{3} u_{max} B H \;,\; A = BH$$

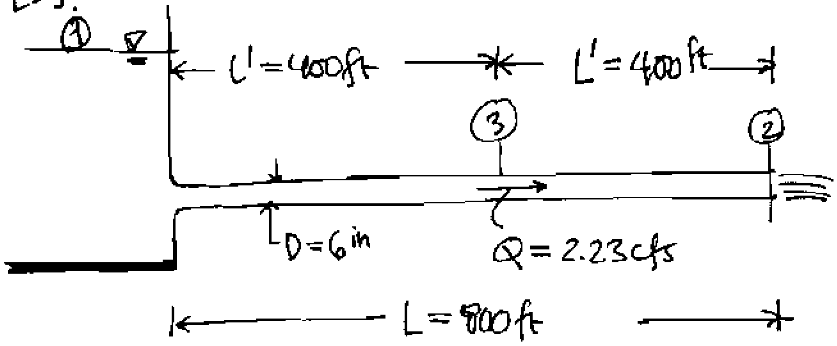
$$V = \frac{Q}{A} = \frac{\frac{1}{3}u_{max}BH}{BH} = \frac{u_{max}}{3}$$

$$\int_A u^3 dA = \int_0^H u_{max}^3 \left(\frac{y}{H}\right)^6 B\,dy = \frac{1}{7} u_{max}^3 B \cdot H.$$

$$\alpha = \frac{1}{AV^3}\int_A u^3 dA = \frac{1}{\cancel{BH}\cdot\left(\frac{u_{max}}{3}\right)^3} \times \frac{1}{7} u_{max}^3 \cancel{BH} = \frac{3^3}{7} = \frac{27}{7} = 3.86 \qquad \boxed{\alpha = 3.86}$$

[3].

$h_L \Rightarrow 6.20\ \text{ft}/100\ \text{ft}$



point ①
$P_1 = 0$
$V_1 = 0$
$z_1 = ?$

point ②
$P_2 = 0$
$V_2 = 11.36\ ft/s$
$z_2 = 0$

point ③
$P_3 = ?$
$V_3 = 11.36\ ft/s$
$z_3 = 0$

CONTINUITY: $V_2 = V_3 = \frac{4Q}{\pi D^2} = \frac{4 \times 2.23\ ft^3/s}{\pi \times (6/12\ ft)^2} = 11.36\ ft/s$

ENERGY ③ - ②

$$\frac{P_3}{\gamma} + z_3 + \frac{V_3^2}{2g} - h_{L_{3\text{-}2}} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

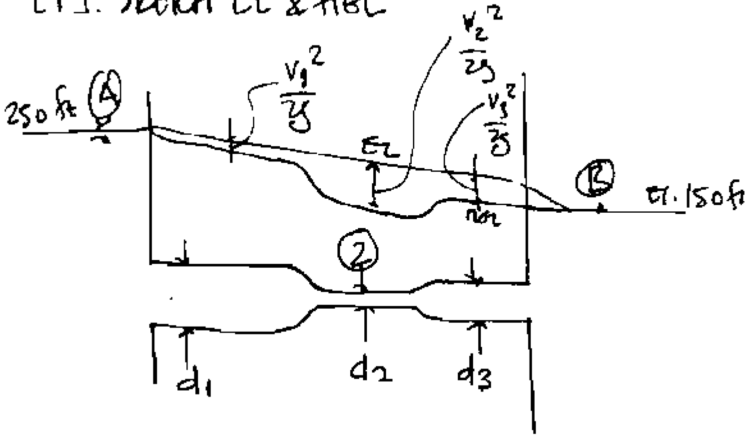
with $h_{L_{3\text{-}2}} = \frac{6.20\ ft}{100\ ft} \times 400\ ft = 24.8\ ft$

$$\frac{P_3}{\gamma} + 0 + \cancel{\frac{11.36^2}{2 \times 32.2}} - 24.8 = 0 + 0 + \cancel{\frac{11.36^2}{2 \times 32.2}}$$

$$\frac{P_3}{\gamma} = 24.8 \Rightarrow P_3 = 24.8\ ft \times 62.4\ \frac{lb}{ft^3} = 1547.52\ \frac{lb}{ft^2} = \frac{1547.52}{144}\ psi$$

$$\boxed{P_3 = 10.75\ psi}$$

[4]. SKETCH EL & HGL



point (A)	point (B)
$z_A = 250$ ft	$z_B = 150$ ft
$P_A = 0$	$P_B = 0$
$V_A = 0$	$V_B = 0$

[5]. $P_2 = ?$ if $d_1 = 1$ ft, $d_2 = 0.25$ ft, $d_3 = 0.50$ ft, $h_L = \frac{V_3^2}{2g}$

ENERGY (A)-(B): $\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} - h_L = \frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g}$

$$0 + 250 + 0 - \frac{V_3^2}{2g} = 0 + 150 + 0$$

$$\Rightarrow \frac{V_3^2}{2g} = 250 - 150 = 100 \Rightarrow V_3^2 = 2 \times 32.2 \frac{ft}{s^2} \times 100 ft = 6440\ ft^2/s^2$$

$$\underline{V_3 = 80.25\ ft/s}$$

$$Q = V_3 \cdot \frac{\pi d_3^2}{4} = \left(80.25 \frac{ft}{s}\right) \times \frac{\pi \times (0.50\ ft)^2}{4} = 15.76\ ft^3/s$$

$$V_2 = \frac{4Q}{\pi d_2^2} = \frac{4 \times 15.76\ ft^3/s}{\pi \times (0.25\ ft)^2} = 321.06\ ft/s$$

ENERGY (A)-(2): $\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$

$$0 + 250^{ft} + 0 = \frac{P_2}{\gamma} + 50 ft + \frac{321.06^2\ ft^2/s^2}{2 \times 32.2\ ft/s^2}$$

$$250 = \frac{P_2}{\gamma} + 50 + 1600.61$$

$$\frac{P_2}{\gamma} = -1400.61 ft \Rightarrow P_2 = -1400.61 \times 62.4 = -87398.06$$

$$\boxed{P_2 = -\frac{87398.06}{144} = -6242.72\ psi}$$