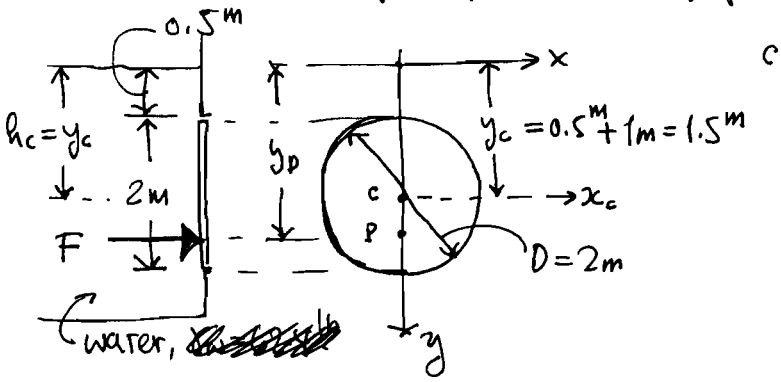
3.7.6. A plane surface is circular with a diameter of 2m. If it is vertical and the top edge is 0.5m below the water surface, find the magnitude of the force on one side and the depth of the center of pressure.



C – CENTROID P – CENTER OF PRESSURE

From Table A.7, p. 738

$$I_c = \frac{\pi D^4}{64} = \frac{\pi (2m)^4}{64} = \frac{\pi}{4} m^4 = 0.7854 m^4$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (2m)^2}{4} = \pi m^2 = 3.1416 m^2$$

Pressure at centroid, $p_c = \gamma h_c = \rho g h_c$, Force: $F = p_c A = \rho g h_c \cdot \frac{\pi D^2}{4}$

Water, $\rho = 1000 \, kg/m^3$

$$F = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(1.5^m) \cdot \frac{\pi \cdot (2m)^2}{4} =$$

CENTER OF PRESSURE:

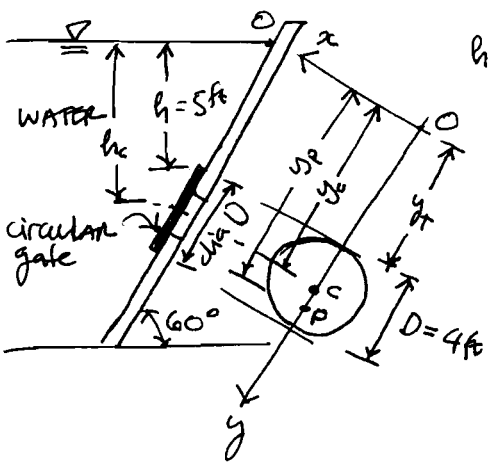
$$y_p = y_c + \frac{I_c}{y_c A} = 1.5^m + \frac{0.7854 m^4}{1.5^m \times 3.1416 m^2}$$

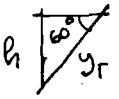
$$y_p = 1.67^m$$

$$F = 3817.03^N$$

$$F = 3.82 \, kN$$

3.7.7. Find the magnitude and depth of the point of application of the force on the circular gate shown in Fig. X3.7.7. if h = 5 ft and D = 4 ft diameter.





$h = y_T \sin 60° \Rightarrow y_T = \frac{h}{\sin 60°} = \frac{5 ft}{\sin 60°} = 5.77 ft$

LOCATION OF CENTROID, $y_c = y_T + \frac{D}{2} = 5.77 ft + \frac{4 ft}{2} = 7.77 ft$

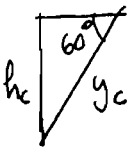
FROM TABLE A.7, p. 758

$$A = \frac{\pi D^2}{4} = \frac{\pi (4 ft)^2}{4} = 4\pi \, ft^2 = 12.57 \, ft^2$$

$$I_c = \frac{\pi D^4}{64} = \frac{\pi \cdot (4 ft)^4}{64} = 4\pi \, ft^4 = 12.57 ft^4$$

Force, $F = p_c A = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(6.73 ft)(12.57 ft^2)$

$$\boxed{F = 5278.80 \, lb}$$

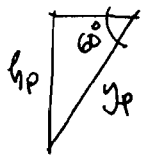


$h_c = y_c \sin 60°$
$= 7.77 ft \sin 60°$
$= 6.73 ft$

LOCATION OF CENTER OF PRESSURE

$$y_p = y_c + \frac{I_c}{y_c A} = 7.77 ft + \frac{12.57 ft^4}{(7.77 ft)(12.57 ft^2)}$$

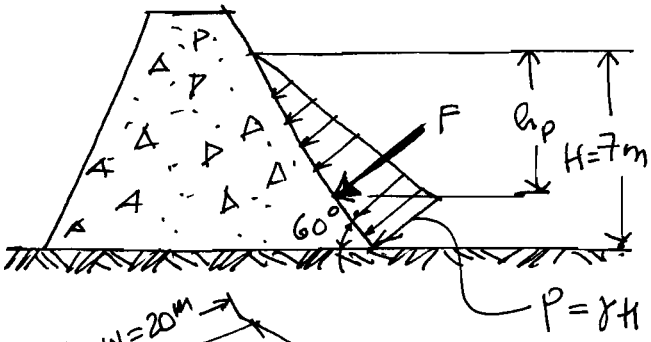
$$y_p = 7.90 ft$$



$$h_p = y_p \sin 60° = (7.90 ft) \sin 60°$$

$$\boxed{h_p = 6.84 ft}$$

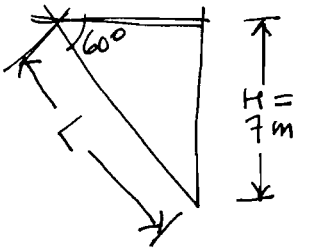
3.24 (Schaum) A dam 20 m long retains 7m of water, as shown in the figure. Find the total resultant force acting on the dam and the location of the center of pressure.

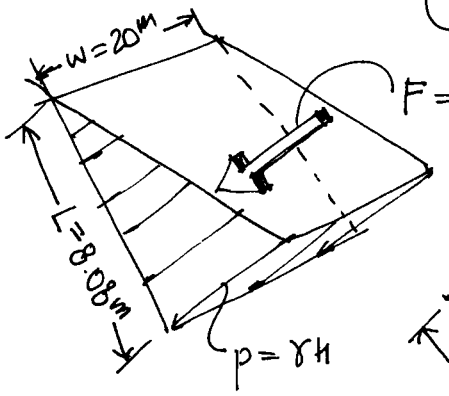


$H = L\sin 60° \Rightarrow L = \frac{H}{\sin 60°}$

$L = \frac{7m}{\sin 60°}$

$L = 8.08\,m$

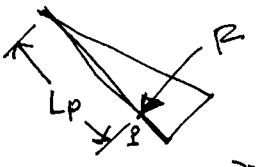




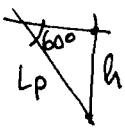
F = volume of pressure prism $= \frac{1}{2}(L)(\gamma H)(w) = \frac{1}{2}\gamma L H w$

$= \frac{1}{2}\left(9810\frac{N}{m^3}\right)(8.08m)(7m)(20m)$

$= 5548536\,N = 5.55\,MN$

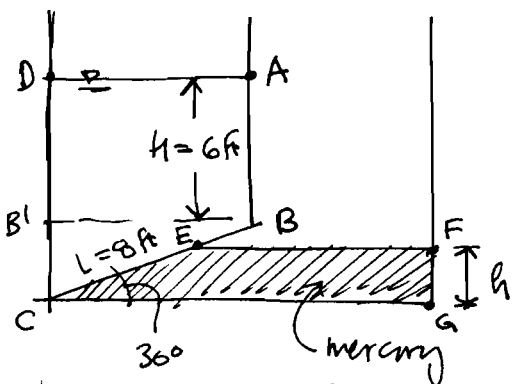


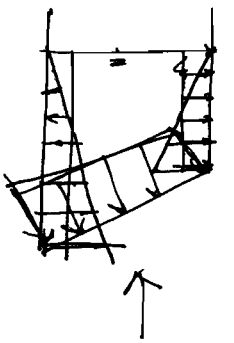
$L_p = \frac{2}{3}L = \frac{2}{3}\times 8.08^m = 5.39\,m$

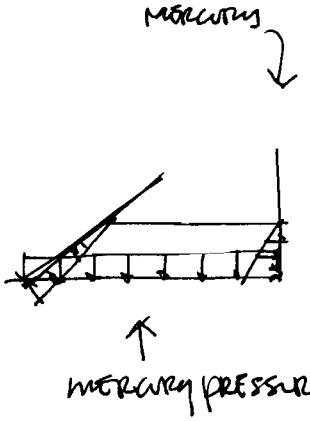


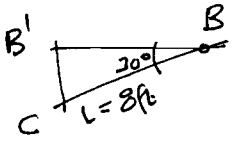
$h_p = L_p \sin 60° = 5.39^m \sin 60° = 4.67^m$

3.44. The tank shown in the figure is 10 ft long, and sloping bottom BC is 8 ft wide. What depth of mercury will cause the resultant moment about C due to the liquids to be 101,300 ft-lb clockwise?



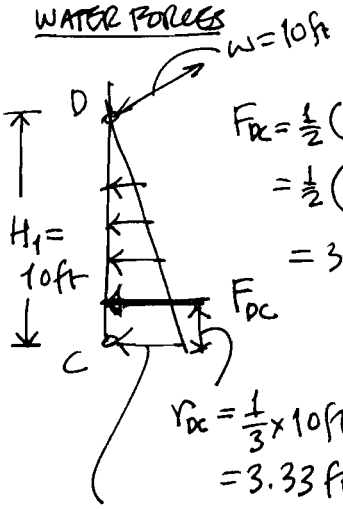






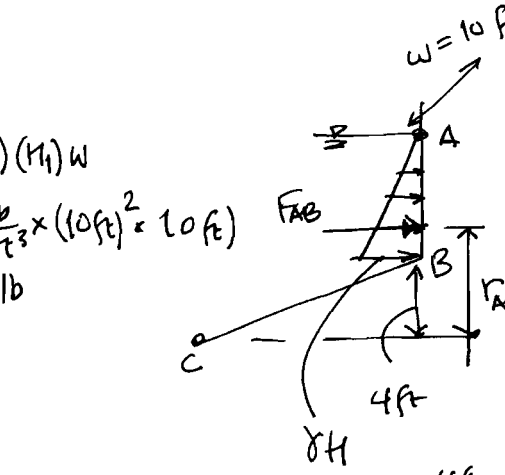
$\overline{B'C} = L\sin 30° = 8ft \times 0.5 = 4ft$

WATER FORCES



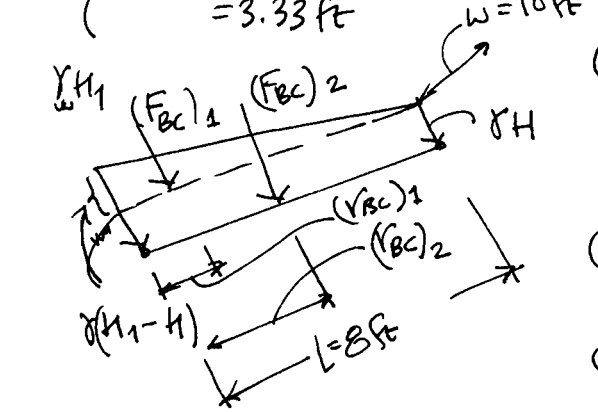
$$F_{DC} = \frac{1}{2}(\gamma_w H_1)(H_1)w = \frac{1}{2}\left(62.4\frac{lb}{ft^3} \times (10ft)^2 \times 10ft\right) = 31200\ lb$$

$$r_{DC} = \frac{1}{3} \times 10ft = 3.33\ ft$$



$$F_{AB} = \frac{1}{2}\gamma H \cdot H \cdot w = \frac{1}{2}\left(62.4\frac{lb}{ft^3}\right)(6ft)^2(10ft) = 11232\ lb$$

$$r_{AB} = 4ft + \frac{1}{3}(6ft) = 6ft$$



$$(F_{BC})_1 = \frac{1}{2}\gamma(H_1 - H)LW = \frac{1}{2}\left(624\frac{lb}{ft^3}\right)(4ft)(8ft)(10ft) = 9984\ lb$$

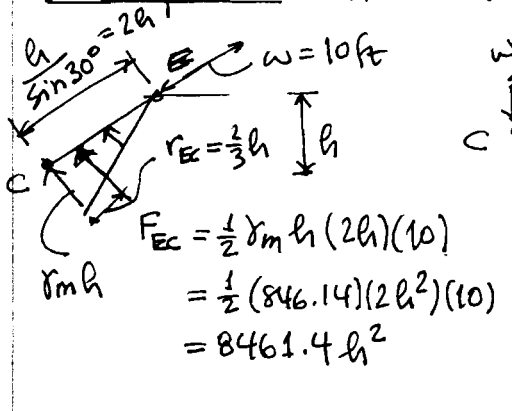
$$(r_{BC})_1 = 1/3(8ft) = 2.67\ ft$$

$$(F_{BC})_2 = \gamma H \cdot L \cdot w = \left(62.4\frac{lb}{ft^3}\right)(6ft)(8ft)(10ft) = 29952\ lb$$

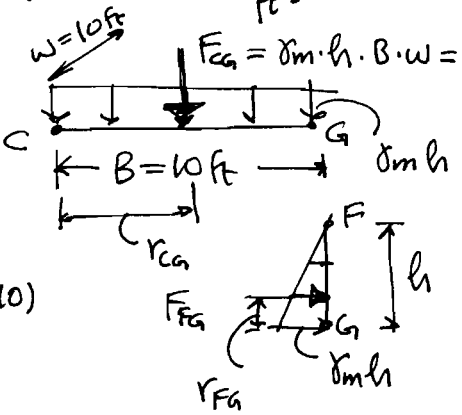
$$(r_{BC})_2 = 1/2(8ft) = 4ft$$

MERCURY FORCES

$$\gamma_m = s_m \gamma_w = 13.56 \times 62.4\frac{lb}{ft^3} = 846.14\ lb/ft^3$$



$$F_{EC} = \frac{1}{2}\gamma_m h(2h)(10) = \frac{1}{2}(846.14)(2h^2)(10) = 8461.4h^2$$



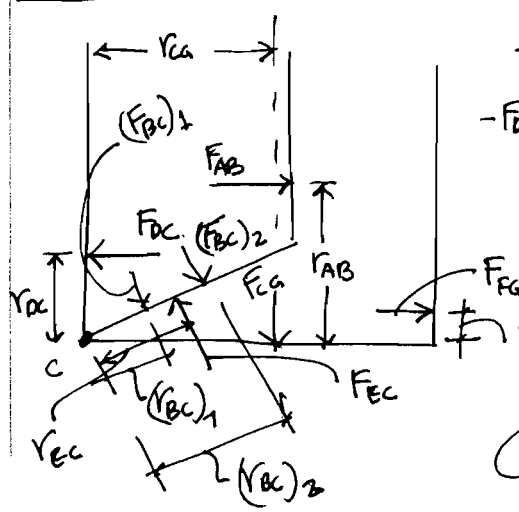
$$F_{CG} = \gamma_m \cdot h \cdot B \cdot w = 846.14 \cdot h \cdot 10 \cdot 10 = 8461.4h$$

$$r_{CG} = \frac{B}{2} = 5\ ft$$

$$F_{FG} = \frac{1}{2}\gamma_m h \cdot h \cdot w = \frac{1}{2}\left(846.14\frac{lb}{ft^3}\right)h^2 \cdot 10\ ft = 4230.7h^2$$

$$r_{FG} = 1/3\,h = h/3$$

FORCES:



$$+\circlearrowleft \Sigma M_C = 101300\ ft\text{-}lb$$

$$-F_{DC}r_{DC} + (F_{BC})_1(r_{BC})_1 + (F_{BC})_2(r_{BC})_2 + F_{AB} \cdot r_{AB} - r_{EC}F_{EC} + r_{CG}F_{CG} + r_{FG} \cdot F_{FG} = 101300$$

$$-31200 \times 3.33 + 9984 \times 2.67 + 29952 \times 4 + 11232 \times 6 - \frac{2}{3}h \cdot 8461.4h^2 + 8461.4h \times 5 + h/3 \times 4230.7h^2 = 101300$$

$$109961.28 - 4230.43h^3 + 42307h = 101300$$

$$\boxed{4230.43h^3 - 42307h - 8661.28 = 0}$$

solve, $h = -2.047 \times 10^{-2}\ ft$, $\boxed{h = 10.01\ ft}$, $h = -9.99\ ft$.