10.6. Most Efficient Cross Section

- Given S_0 and n, $V \sim R_h \Rightarrow$ for A = constant, V is max when P is minimum
  $\Rightarrow$ most efficient x-section (hydraulic efficiency).

- Of all geometric figures, the circle has the least perimeter for a given area.
  What is the most efficient circular x-section? $\Rightarrow$ maximize R_h with A=const.

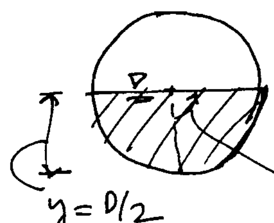
$$A = \frac{D^2}{4}(\theta - \sin\theta\cos\theta) \Rightarrow D = \frac{2\sqrt{A}}{(\theta - \sin\theta\cos\theta)^{1/2}}$$

$$P = \theta \cdot D = \frac{2\sqrt{A}\,\theta}{\sqrt{\theta - \sin\theta\cos\theta}}, \quad R_h = \frac{A}{P} = \frac{\sqrt{A}\sqrt{\theta - \sin\theta\cos\theta}}{2\theta}$$

$$\frac{dR_h}{d\theta} = 0 \Rightarrow \frac{\sqrt{A}}{4\theta^2\sqrt{\theta - \sin\theta\cos\theta}}\left[-\theta + 2\sin\theta\overset{\cos\theta}{} - \theta\cos^2\theta + \theta\sin^2\theta\right] = 0$$

$$\Rightarrow -\theta + \underbrace{2\sin\theta\cos\theta}_{\sin 2\theta} - \theta\underbrace{(\cos^2\theta - \sin^2\theta)}_{\cos 2\theta} = 0 \Rightarrow -\theta + \sin 2\theta - \theta\cos 2\theta = 0$$

solutions to $\sin 2\theta = \theta(1 + \cos 2\theta) \Rightarrow \begin{cases} \theta = \pi/2 \\ \theta = 0 \\ \boxed{\theta = \pi/2} \end{cases}$

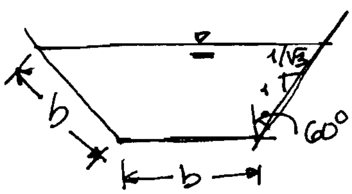


For $\theta = \pi/2$, $P = \theta \cdot D = \frac{\pi D}{2}$

$$A = \frac{D^2}{4}(\theta - \sin\theta\cos\theta) = \frac{D^2}{4}\left(\frac{\pi}{2} - \cancel{\sin\frac{\pi}{2}\cos\frac{\pi}{2}}^{\,0}\right) = \frac{\pi D^2}{8}, \quad \boxed{R_h = \frac{A}{P} = \frac{\pi D^2/8}{\pi D/2} = \frac{D}{4} = \frac{y}{2}}$$

For a trapezoidal channel (see development in pp. 423-424)

$\rightarrow$ Most efficient x-section $\Rightarrow m = 1/\sqrt{3}$



$\leftarrow$ half octagon, also $D_h = \frac{y}{2}$

10.7 Circular section (again) — read pp 426–428

$$\boxed{q = \frac{g}{\nu}\frac{y_0^3}{3}S}$$