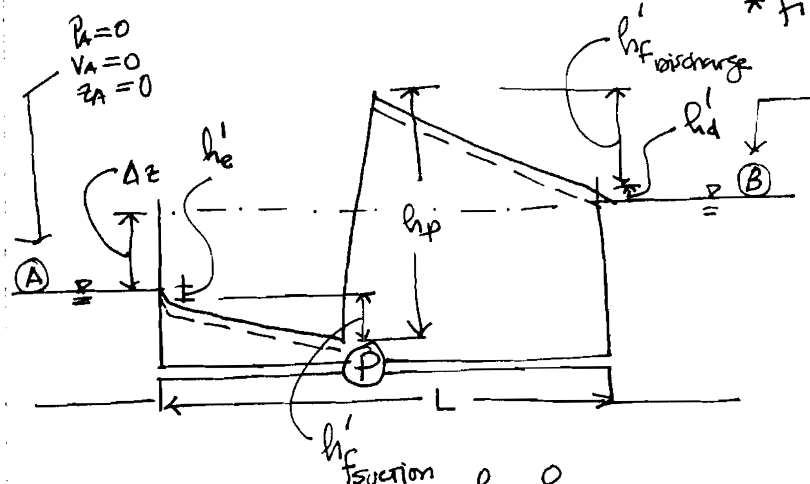
Pipeline with pump

* Simultaneous solution of two equations
  - a system equation — ENERGY EQUATION
  - a pump equation — EITHER
    - ▸ information on power, $P=\gamma Q h_p$
    - ▸ Pump equation
      * provided by manufacturer
      * fitted from experiments



ENERGY EQUATION (A)-(B) $\Rightarrow$

$$\cancelto{0}{\frac{p_A}{\gamma}}+\cancelto{0}{z_A}+\cancelto{0}{\frac{V_A^2}{2g}}-h_f-\Sigma h'+h_p=\cancelto{0}{\frac{p_B}{\gamma}}+\cancelto{\Delta z}{z_B}+\cancelto{0}{\frac{V_B^2}{2g}}$$

$$h_p=\Delta z+\underbrace{h_f+\Sigma h'}_{h_L=\text{TOTAL LOSS}} \Rightarrow \underline{h_p=\Delta z+h_L}$$

with $h_f=f\frac{L}{D}\frac{V^2}{2g}=\frac{8fLQ^2}{\pi^2 g D^5}$, and $\Sigma h'=\Sigma K\cdot\frac{V^2}{2g}=\Sigma K\cdot\frac{8Q^2}{\pi^2 g D^4}$,

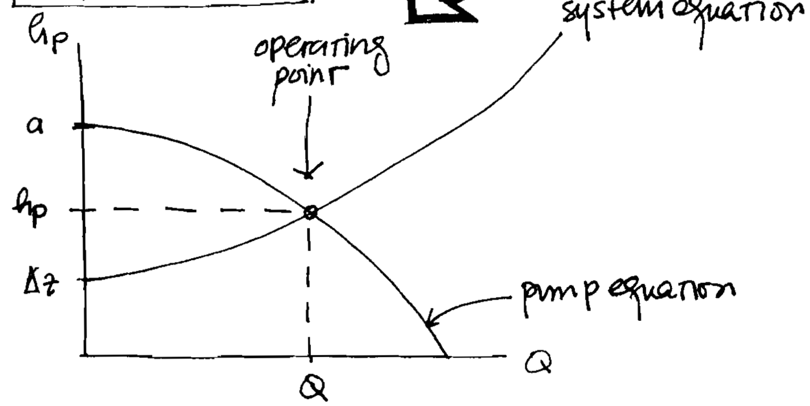
$$h_p=\Delta z+\frac{8fLQ^2}{\pi^2 g D^5}+\frac{8Q^2\cdot\Sigma K}{\pi^2 g D^4} \Rightarrow \boxed{h_p=\Delta z+\frac{8Q^2}{\pi^2 g D^4}\left(f\frac{L}{D}+\Sigma K\right)}$$ ← system equation

pump equation: $\boxed{h_p=a+bQ+cQ^2}$ ← For centrifugal pumps

Alternative ~~information~~ equation

$$\boxed{P=\gamma Q h_p}$$

Graphical solution



<u>Example</u>. $\nu = 1\times10^{-5}\,m^2/s$, $g = 9.806\,m/s^2$, $e = 0.0001\,m$, $D = 2m$, $L = 100m$, $\Delta z = 20m$, $\Sigma K = 2.5$, $a = 60$, $b = 0$, $c = -0.012$

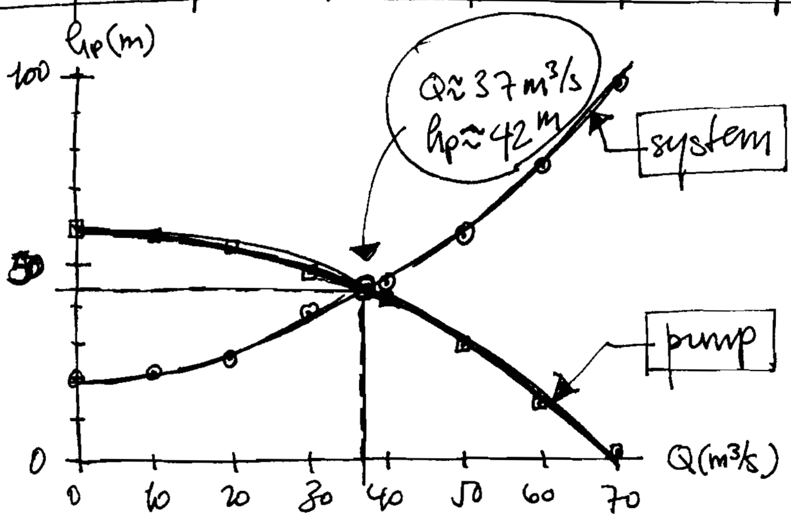
$$\frac{e}{D} = 0.00005$$

(A) $$h_p = \Delta z + \frac{8Q^2}{\pi^2 g D^4}\left(f\frac{L}{D} + \Sigma K\right) = 20 + \frac{8\cdot Q^2}{\pi^2\times 9.806\times 2^4}\left(f\frac{100}{2} + 2.5\right)$$

$$h_p = 20 + 0.0052Q^2(50f + 2.5) \quad \text{system}$$

(B) $$h_p = a + bQ + cQ^2 = 60 + 0\cdot Q - 0.012Q^2 = 60 - 0.012\,Q^2, \text{ pump}$$

	$Q(m^3/s)$	0.00	10.0	20.0	30.0	40.0	50.0	60.0	70.0
(S)	$h_p(m)$	20	21.65	26.48	34.47	45.62	59.92	77.37	97.97
(P)	$h_p(m)$	60	58.8	55.20	49.20	40.8	30.00	16.80	1.20



$$\mathbb{R} = \frac{4Q}{\pi\nu D} = \frac{4Q}{\pi\cdot10^{-5}\cdot 2}$$

$$\mathbb{R} = 6.4\times10^4 Q$$

$$Q = \sqrt{\frac{60 - h_p}{0.012}}$$

<u>ITERATIVE PROCEDURE</u>: assume Q, find $f\,(e/D, 4Q/\pi\nu D)$, calculate h_p (A) and recalculate Q from (B) using fHA for f

- assume $Q = 60\,m^3/s$, $\mathbb{R} = 3.84\times10^6$, $f = 0.0112$, $h_p = 77.28^m$, $Q = 37.94i$ ← imaginary

$\bar{Q} = 36.43\frac{m^3}{s}$ $Q = 20\,m^3/s$, $\mathbb{R} = 1.28\times10^6$, $f = 0.0122$, $h_p = 26.47^m$, $Q = 52.86\,m^3/s$

$Q = 52.86\,m^3/s$, $\mathbb{R} = 3.38\times10^6$, $f = 0.0131$, $h_p = 65.84^m$, $Q =$ imaginary

$\bar{Q} = 38.07\frac{m^3}{s}$ $Q = 30\,m^3/s$, $\mathbb{R} = 1.92\times10^6$, $f = 0.0118$, $h_p = 34.46^m$, $Q = 46.13\,m^3/s$

$\bar{Q} = 38.03\frac{m^3}{s}$ $Q = 36\,m^3/s$, $\mathbb{R} = 2.30\times10^6$, $f = 0.0115$, $h_p = 40.75^m$, $Q = 40.05\,m^3/s$

$\bar{Q} = 37.74\,m^3/s$ $Q = 38.03\,m^3/s$, $\mathbb{R} = 2.43\times10^6$, $f = 0.0115$, $h_p = 43.18^m$, $Q = 37.44\,m^3/s$

$\bar{Q} = 37.80\,m^3/s$ $Q = 37.74\,m^3/s$, $\mathbb{R} = 2.42\times10^6$, $f = 0.0116$, $h_p = 42.81^m$, $Q = 37.85\,m^3/s$

$Q = 37.80\,m^3/s$, $\mathbb{R} = 2.42\times10^6$, $f = 0.0116$, $h_p = 42.88\,m$, $Q = 37.77\,m^3/s$

DIRECT SOLUTION USING NUMERICAL SOLVER IN CALCULATORS

Make (A) = (B), i.e.,

$$60 - 0.012Q^2 = 20 + 0.0052 \cdot Q^2 \cdot (50 \cdot f + 2.5)$$

replacing f with $fHA\left(\frac{ee}{D}, \frac{4Q}{\pi \cdot Nu \cdot D}\right)$ or $fSJ\left(\frac{ee}{D}, \frac{4Q}{\pi \cdot Nu \cdot D}\right)$, i.e.,

'60 − 0.012*Q^2 = 20 + 0.0052*Q^2*(50*fHA(ee/D, 4*Q/(π*Nu*D)) + 2.5)'

Solving for Q, $\boxed{Q = 37.79\ m^3/s}$

$$\boxed{h_p = 60 - 0.012 \times 37.79^2 = 42.86\ m}$$