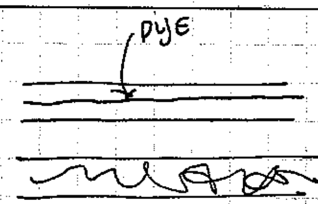
8.9. Turbulent Flow

laminar flow: particles move in straight lines

Turbulent flow: particles follow random paths

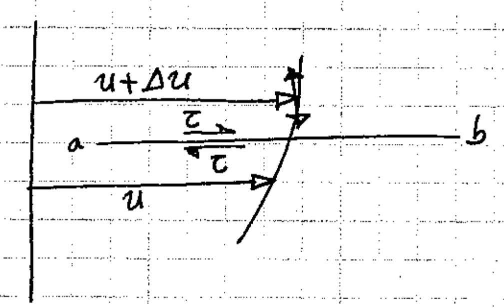
FLOW VISUALIZATION

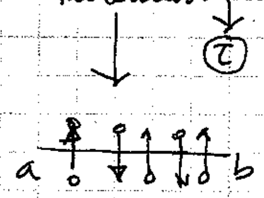


VELOCITY DISTRIBUTION AND MOMENTUM EXCHANGE

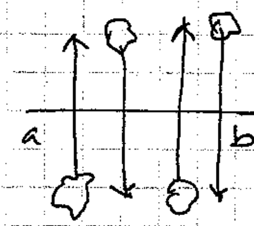
layers exchange molecules → τ

Transfer of finite masses across ab → τ





o molecule



finite fluid mass or eddy

Turbulent flow:
- velocity at a point fluctuates in both magnitude and direction
- fluctuations result from a multitude of small eddies created by viscous shear between adjacent particles
- eddies grow in size and dissapear as its particles merge into adjacent eddies

Expressions for turbulent shear stress

- First expression: assume mechanism similar to molecular exchange of momentum but involving eddies

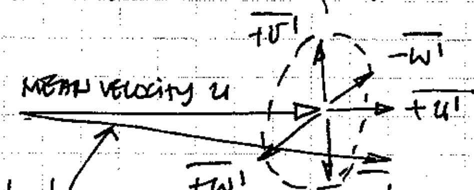
$$\tau_t = \eta \frac{du}{dy}, \quad \eta = \text{non-constant eddie viscosity}$$

$$\varepsilon = \frac{\eta}{\rho} = \text{kinematic eddy viscosity}$$

Total shear $\quad \tau = \tau_\ell + \tau_t = \mu\frac{du}{dy} + \eta\frac{du}{dy} = \rho(\nu+\varepsilon)\frac{du}{dy}$ (A)

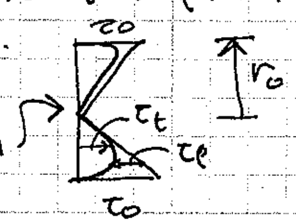
Usually $\tau_t >> \tau_\ell$ except near the solid boundaries where $v'=0$ (no flow accross boundary). Since τ_t is related to $u'v'$ (see below), $\tau_t \approx 0$ near the boundary and $\tau = \tau_\ell = \mu\frac{du}{dy}$. away from the wall, in turbulent flow, $du/dy \to 0$, but η could be large, that's where $\tau_t >> \tau_\ell$. at the ℄ of the pipe $\tau = 0$.

VELOCITIES IN TURBULENT FLOW

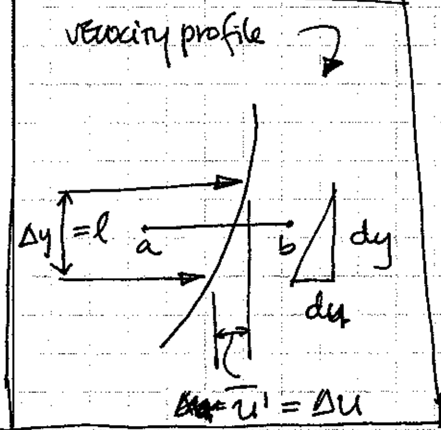


instantaneous velocity $= \sqrt{(u \pm u')^2 + (v')^2 + (w')^2}$

shear stress distribution



- Second expression



$|\overline{u'}|$ = mean x-velocity fluctuation

select $\Delta y = l$ = mixing length so that $\Delta u = |\overline{u'}|$

shear force on plane ab is calculated using momentum principle

$$F = \tau dA = \rho Q \Delta V = \rho(\underset{\uparrow\, Q}{v' dA})(\underbrace{u+u'-u}_{\Delta V}) = \rho u' v' dA$$

averaging over time $\quad \tau = \dfrac{F}{dA} = \underbrace{-\rho\overline{u'v'}}_{\text{Reynolds stress}}$

Prandtl reasoned that $|\overline{v'}| = C|\overline{u'}|$, and introduced the concept of a mixing length l as the distance perpendicular to the flow direction such that $\Delta u = |\overline{u'}|$. From the figure it follows that $\Delta u = l \cdot (du/dy)$, thus $|\overline{u'}| = l\, du/dy$, and $|v'| = C l\, du/dy$. Prandtl showed that $-\overline{u'v'} \sim l^2 \left(\frac{du}{dy}\right)^2$

and

$$\tau = -\rho\overline{u'v'} = \rho l^2 \left(\frac{du}{dy}\right)^2 \qquad (B)$$

ADDITIONAL INFORMATION: von Karman found that, near a solid boundary, $l = \kappa y$, where y is the distance from the wall. The value κ (kappa) is known as the von Karman's constant. For water, $\kappa = 0.40$.

For example on a shallow turbulent channel flow, assuming that the shear stress through the flow is constant and equal to the wall shear stress, i.e., $\tau = \tau_0$, with $l = \kappa y$, (B) becomes

$$\tau_0 = \rho \kappa^2 y^2 \left(\frac{du}{dy}\right)^2 \Rightarrow \frac{du}{dy} = \frac{\tau_0}{\rho} \cdot \frac{1}{\kappa y} \qquad (C)$$

The quantity $\sqrt{\frac{\tau_0}{\rho}} = u^*$ is referred to as a shear velocity. It is not a velocity that you can measure in the flow, it just happens that $\sqrt{\tau_0/\rho}$ has units of velocity, and, thus, it's given the symbol u_*. Thus, (C) becomes

$$\frac{du}{dy} = \frac{u_*}{\kappa y} \Rightarrow du = \frac{u_*}{\kappa}\frac{dy}{y}, \text{ integrating } \quad \underline{u = \frac{u_*}{\kappa}\ln y + C}$$

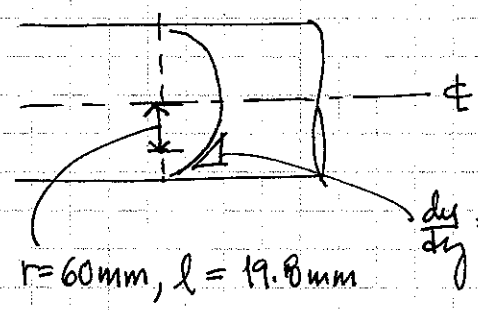
Thus, turbulent flow can produce a logarithmic velocity distribution

(see section 8.1)

EXERCISES

8.9.2. Water at 20°C flows through a 240-mm-diameter pipe. Tests have determined that at a distance of 60 mm from the pipe centerline the mixing length l is 19.8 mm and from the velocity profile $du/dy = 5.33\ s^{-1}$. Find at that radius (a) the total shear stress, (b) the eddy viscosity, (c) the viscous shear, and (d) the turbulent shear.

• Table A-1, for $T = 20°C \Rightarrow \rho = 998.2\ kg/m^3,\ \mu = 0.001002\ N.s/m^2$



(a) Eq.(8.34) $\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

$$\tau = \left(998.2\ \frac{kg}{m^3}\right)(0.0198\,m)^2 \cdot \left(5.33\ s^{-1}\right)^2$$

$$\tau = 11.12\ \frac{kg \cdot m/s^2}{m^2} = 11.12\ \frac{N}{m^2}$$

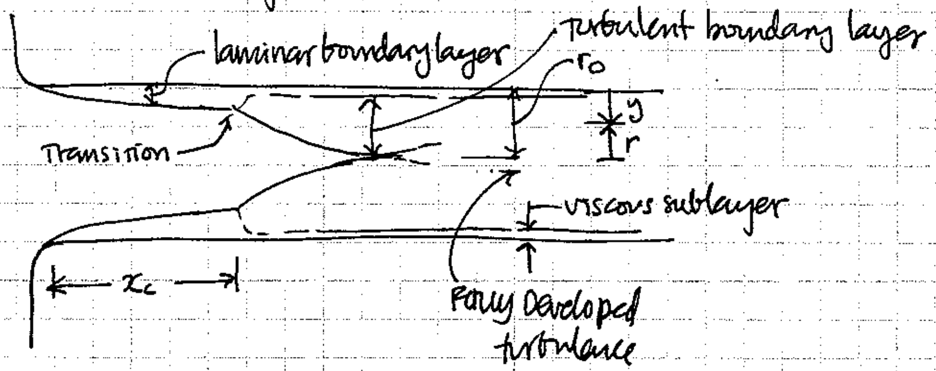
(b) Eq.(8.32) $\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy} \Rightarrow \eta \frac{du}{dy} = \tau - \mu \frac{du}{dy} \Rightarrow \eta = \frac{\tau}{du/dy} - \mu$

$$\eta = \frac{11.12\ N/m^2}{5.33\ s^{-1}} - 0.001002\ \frac{N.s}{m^2} = 2.084810\ \frac{N.s}{m^2}$$

(c) $\tau_l = \mu \frac{du}{dy} = \left(0.001002\ \frac{N.s}{m^2}\right)\left(5.33\ s^{-1}\right) = 0.005341\ N/m^2$

(d) $\tau_t = \eta \frac{du}{dy} = \left(2.084810\ \frac{N.s}{m^2}\right)\left(5.33\ s^{-1}\right) = 11.112037\ N/m^2$

8.10. Viscous sublayer in turbulent flow



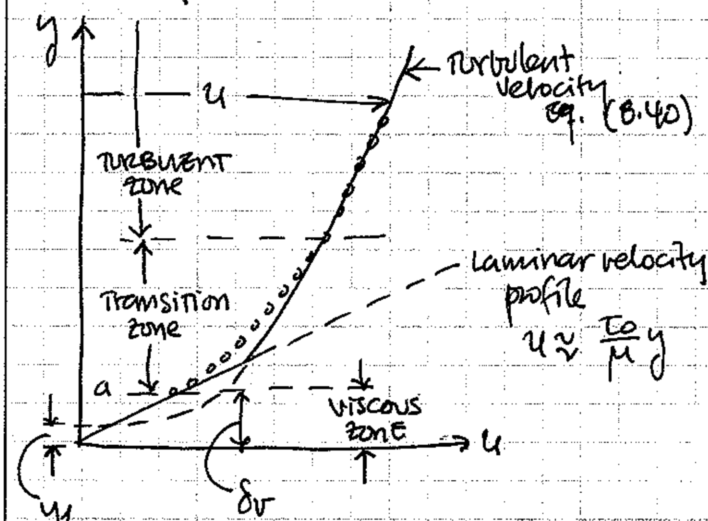
Flow with $\mathbb{R} > 4000$ will be turbulent, but near the entrance a laminar boundary layer develops and grows in thickness until reaching a critical distance x_c at which point the boundary layer becomes turbulent. The turbulent boundary layer grows faster until turbulent flow fills the full pipe. The conditions for transition from laminar to turbulent boundary layer are given by

$$\mathbb{R}_x = \frac{Ux}{\nu} \approx 5\times 10^5$$

Fully turbulent flow is developed within 20 to 40 pipe diameters.

Near the wall, in turbulent flow, there exist a very thin layer where viscous stresses are dominant — THE LAMINAR SUBLAYER or VISCOUS SUBLAYER

VELOCITY PROFILE IN TURBULENT FLOW $\longrightarrow$ $u_* = \sqrt{\tau_0/\rho}$ = shear velocity



In the viscous sublayer

$$\tau_0 = \mu \frac{u}{y} \Rightarrow \frac{\tau_0}{\rho} = \frac{\nu u}{y}$$

$$\Rightarrow \frac{u\nu}{y} = u_*^2 \Rightarrow \boxed{\frac{u}{u_*} = \frac{y u_*}{\nu}}$$

law of the wall

valid for $0 \le y u_*/\nu \le 5$

Thickness of the viscous sublayer

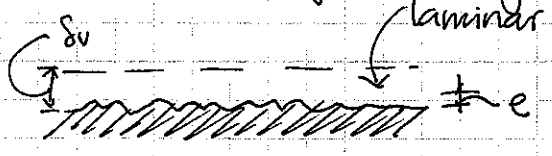
$$\boxed{\delta_v = \frac{5\nu}{u_*}}$$

From $\tau_0 = \frac{f}{8}\rho V^2 \Rightarrow \frac{\tau_0}{\rho} = \frac{f}{8}V^2 \Rightarrow u_*^2 = V^2 \cdot \frac{f}{8} \Rightarrow \boxed{u_* = V\sqrt{f/8}}$, into

$$\boxed{\delta_v = \frac{5\nu}{V\sqrt{f/8}} = \frac{14.14\nu}{V\sqrt{f}} = \frac{14.14\cdot D}{\frac{VD}{\nu}\sqrt{f}} = \frac{14.14 D}{\mathbb{R}\sqrt{f}}}$$

roughness of a solid wall

If $\delta_v > e \Rightarrow$ hydraullicaly smooth surface, i.e., $\frac{5\nu}{u_*} > e$, or $\boxed{\frac{e u_*}{\nu} < 5}$



If $e > 14\,\delta_v \Rightarrow$ fully rough flow, i.e., $\boxed{e u_*/\nu > 70}$

In the range $5 \le e u_*/\nu \le 70$ or $\delta_v \le e \le 14\,\delta_v \Rightarrow$ transitionally rough

EXERCISES

8.10.2. Water in a pipe ($f = 0.018$) is at a temperature of $70°F$. (a) If the mean velocity is 14 fps, what is the nominal thickness δ_v of the viscous sublayer? (b) What will δ_v be if we increase the velocity to ~~5 m/s~~ 24 fps and f does not change?

Table A.1, for $T = 70°F$, $\nu = 10.59 \times 10^{-6}\ ft^2/sec$

(a) $V = 14\ ft/s$, $f = 0.018$

$$u_* = V\sqrt{\frac{f}{8}} = \left(14\ \frac{ft}{s}\right)\sqrt{\frac{0.018}{8}} = 0.664\ ft/s$$

$$\delta_v = \frac{5\nu}{u_*} = \frac{5 \times 10.59 \times 10^{-6}\ ft^2/sec}{0.664\ ft/sec} = 7.97 \times 10^{-5}\ ft = 0.0009564\ in$$

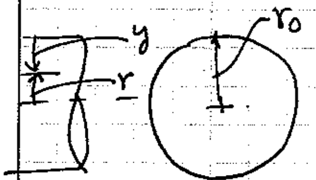
(b) $V = 24\ ft/s$, $f = 0.018$

$$u_* = V\sqrt{\frac{f}{8}} = (24\ ft/s)\sqrt{\frac{0.018}{8}} = 1.138\ ft/s$$

$$\delta_v = \frac{5\nu}{u_*} = \frac{5 \times 10.59 \times 10^{-6}\ ft^2/sec}{1.138\ ft/s} = 4.65 \times 10^{-5}\ ft = 0.000583\ in$$

8.11. Velocity profile in turbulent flow

see pp. 2: $u = \frac{u_*}{\kappa} \ln y + C$, with $\kappa = 0.40$ and $u = u_{max}$ at $y = r_0$



$$\boxed{\frac{u_{max} - u}{u_*} = 2.5 \ln\left(\frac{r_0}{y}\right)} \leftarrow \text{velocity defect law}$$

$y = r_0 - r$, and using log ($\log_{10}$) $\Rightarrow$

$$\boxed{u = u_{max} - 2.5 u_* \ln\left(\frac{r_0}{r_0 - r}\right) = u_{max} - 5.76\, u_* \log\left(\frac{r_0}{r_0 - r}\right)}$$

applicable away from wall and from centerline

- Near the wall, velocity defect law shows that $u = 0$ at a distance $y_1 \neq 0$ $\Rightarrow$ viscous sublayer
- Transition zone $\quad \frac{u}{u_*} = 2.5 \ln\left(\frac{u_* y}{\nu}\right) + 5.0$

Discharge

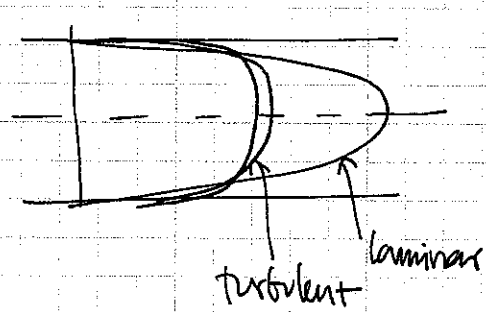
$$Q = \int u\, dA = \int_0^{r_0} \left[u_{max} - 2.5 u_* \ln\left(\frac{r_0}{r_0 - r}\right)\right] \cdot 2\pi r\, dr \Rightarrow V = \frac{Q}{A}$$

$$V = u_{max} - 2.5 u_* \left[\ln r_0 - \frac{2}{r_0^2} \int_0^{r_0} r \ln(r_0 - r)\, dr\right]$$

using $u_* = \sqrt{\frac{\tau_0}{\rho}} = V\sqrt{\frac{f}{8}} \Rightarrow \quad V = u_{max} - 1.326\, V \sqrt{f}$

from which $\Rightarrow$ Pipe factor $= \boxed{\frac{V}{u_{max}} = \frac{1}{1 + 1.326\sqrt{f}}}$

$$\boxed{u = (1 + 1.326\sqrt{f})V - 2.04\sqrt{f}\, V \log\left(\frac{r_0}{r_0 - r}\right)}$$

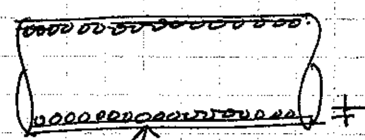


correction factors

- kinetic energy: $\alpha = 1 + 2.7f$
- momentum: $\beta = 1 + 0.98f$

8.12. Pipe Roughness

Nikuradse's experiments



$\frac{e}{D}$ = relative roughness

In general $f = \phi(\mathbb{R}, e/D)$

In Nikuradse's experiments $0.000985 < \frac{e}{D} < 0.0333$

expressions for $f(\frac{e}{D}, \mathbb{R})$

- Smooth-pipe flow, $\delta_v > e$:

$$\boxed{\frac{1}{\sqrt{f}} = 2\log\left(\frac{\mathbb{R}\sqrt{f}}{2.51}\right)} \quad \text{(A)} \leftarrow \text{implicit in } f\text{, requires iterative solution}$$

Coolebrook approximates it by using

$$\boxed{\frac{1}{\sqrt{f}} = 1.8\log\left(\frac{\mathbb{R}}{6.9}\right)} \leftarrow \text{explicit in } f$$

experiments by Blasius, for $3\times10^3 \le \mathbb{R} \le 1\times10^5$, $\boxed{f = \frac{0.316}{\mathbb{R}^{0.25}}}$

corresponding to a velocity distribution $\frac{u}{u_{max}} = \left(\frac{y}{r_0}\right)^{1/7}$ ← seventh-root law

- Fully-rough pipe flow $\delta_v < \frac{e}{14}$ or $e > 14\delta_v$:

$$\boxed{\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7}{e/D}\right)} \quad \text{(B)}$$

- Transitionally rough (all turbulent flow) → combine (A) & (B)

$$\boxed{\frac{1}{\sqrt{f}} = -2\log\left(\frac{e/D}{3.7} + \frac{2.51}{\mathbb{R}\sqrt{f}}\right)}$$

Coolebrook-White equation

NOTE: C-W → (A) if $e = 0$ (smooth)
C-W → (B) if $\mathbb{R} \to \infty$ (fully rough)

- Explicit equations

Haaland: (1983)

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{e/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathbb{R}}\right]$$

Swamee-Jain:
(   )

EXERCISES

8.12.1. Using the implicit equation (8.46), the approximate equation (8.47), and Blasius' equation (8.48), solve for the smooth-pipe friction factor f using Reynolds numbers of (a) 4×10^3, (b) 2×10^4, (c) 1×10^5. (d) for which of these 3 values do the equations show the most variation in f?

(8.46) $\frac{1}{\sqrt{f}} = 2\log\left(\frac{\mathrm{IR}\sqrt{f}}{2.51}\right)$

(8.47) $\frac{1}{\sqrt{f}} = 1.8\log\left(\frac{\mathrm{IR}}{6.9}\right) \Rightarrow f = \frac{1}{\left[1.8\log\left(\frac{\mathrm{IR}}{6.9}\right)\right]}$

(8.48) $f = \frac{0.316}{\mathrm{IR}^{0.25}}$

	(8.46)	(8.47)	(8.48)	
$\mathrm{IR}=4\times10^3$	3.99×10^{-2}	4.04×10^{-2}	3.97×10^{-2}	
$\mathrm{IR}=2\times10^4$	2.59×10^{-2}	2.57×10^{-2}	2.66×10^{-2}	← most variation.
$\mathrm{IR}=1\times10^5$	1.80×10^{-2}	1.78×10^{-2}	1.78×10^{-2}	

8.13. Chart for friction factor (Moody diagram) — see p. 283