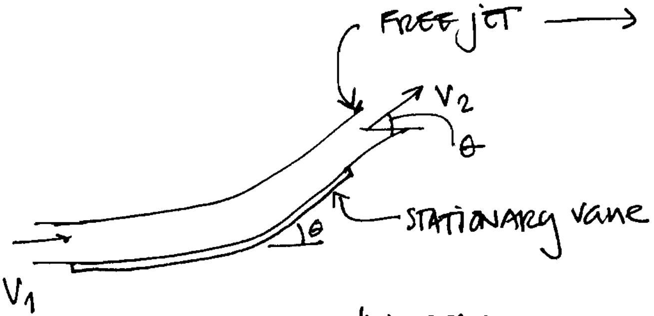
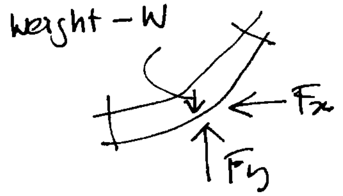
# 6.6. Force of a free jet on a stationary vane or blade

FREE JET ⟶ $p=0$, $pA=0$

Friction forces may not be negligible ⇒ $V_2 < V_1$




Force components of vane on flow




momentum equation in this case:

$$\Sigma F_x = \Delta(\rho Q V)_x : \quad -F_x = \rho Q (V_2 \cos\theta - V_1)$$

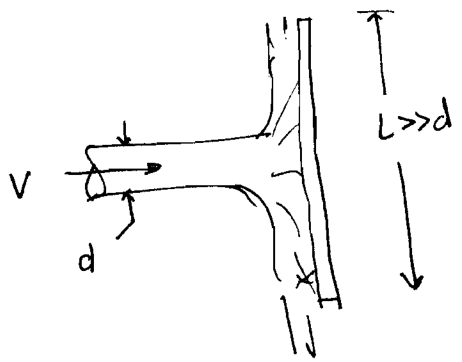
$$\Sigma F_y = \Delta(\rho Q V)_y : \quad F_y - W = \rho Q (V_2 \sin\theta - 0)$$

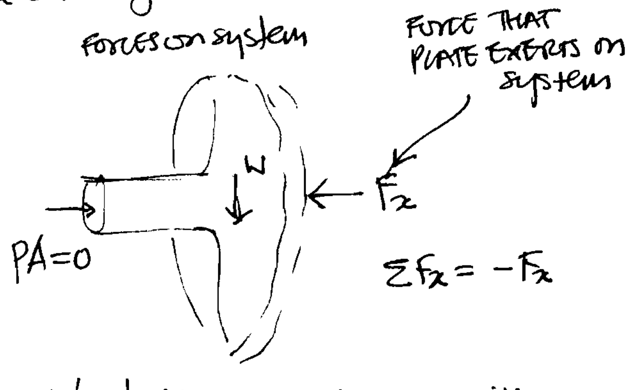
↑ Typically neglected

(SEE Sample Problem 6.3, p. 199)

EXERCISES.

6.6.2. A 40-mm-diameter jet has a velocity of 25 m/s. If this jet were to strike a large plate normally, what would be the resultant force on the jet?

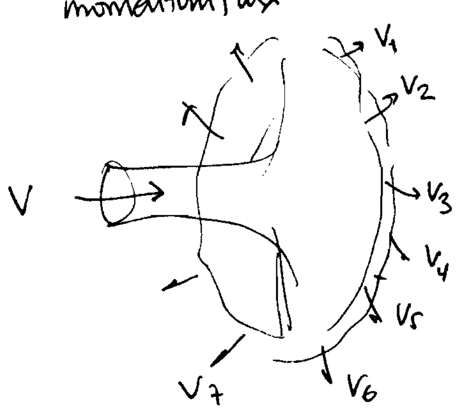



$\Sigma F_x = -F_x$

momentum flux ——— all $V_i$'s $\perp V$ : $\Delta(\rho Q V)_x = \text{OUT} - \text{in}$

$$= 0 - \rho Q V$$



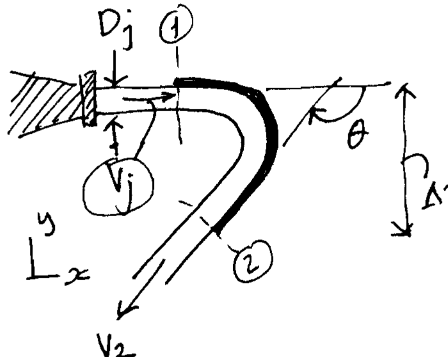

momentum equation: $\Sigma F_x = \Delta(\rho Q V)_x$

$$-F_x = -\rho Q V \Rightarrow F_x = \rho Q V = \rho V \frac{\pi D^2}{4} V$$

$$F_x = \frac{\pi}{4}\rho V^2 D^2 = \frac{\pi}{4} \times 1000\,\frac{kg}{m^3} \times \left(25\,\frac{m}{s}\right)^2 \times (0.04\,m)^2$$

$$\boxed{F_x = 785.4\ N}$$

6.6.6. In Fig X6.66 assume that friction is negligible, that $\theta = 115°$, and that the water jet has a velocity of 95 fps and a diameter of 1 in. Find (a) the component of the force acting on the blade in the direction of the jet; (b) the force component normal to the jet; (c) the magnitude and direction of the resultant force exerted on the blade.

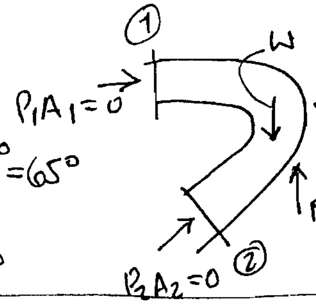



Assuming no friction (and no change in the x-section of the jet, as well as $\Delta z$ negligible) then $V_2 = V_j = 95$ fps

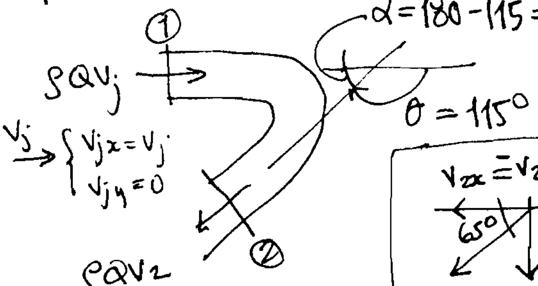
**Forces on system**

$\Sigma F_x = -F_x$

$\Sigma F_y = F_y - W = F_y$ (W negligible)




**momentum flow**




$V_{2x} = V_2 \cos 65° = -95 \times 0.4226 = -40.15$ ft/s

$V_{2y} = V_2 \sin 65° = -95 \times 0.9063 = -86.10$ ft/s

$$\Delta(\rho Q V)_x = \text{out} - \text{in} = \rho Q V_{2x} - \rho Q V_{jx} = \rho Q(-40.15) - \rho Q(95)$$

$$= \frac{62.4}{32.2} \times 0.52(-40.15 - 95) = -136.19 \text{ lb}$$

$$Q = V_j \cdot \frac{\pi D^2}{4} = 95 \times \frac{\pi \times}{4}\left(1/12\right)^2 = 0.52 \text{ cfs}$$

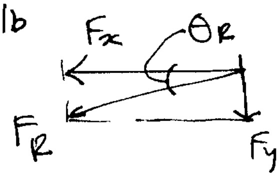
$$\Delta(\rho Q V)_y = \text{out} - \text{in} = \rho Q(V_{2y} - V_{1y}) = \rho Q(-86.10 - 0) = \frac{62.4}{32.2} \times 0.52 \times (-86.10)$$

$$= -86.76 \text{ lb}$$

momentum eqn:

$\Sigma F_x = \Delta(\rho Q V)_x \Rightarrow -F_x = -136.19 \text{ lb} \Rightarrow \boxed{F_x = 136.19 \text{ lb} \leftarrow}$

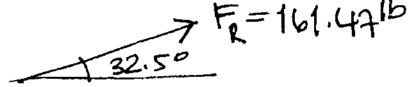
$\Sigma F_y = \Delta(\rho Q V)_y \Rightarrow F_y = -86.76 \text{ lb} \Rightarrow \boxed{F_y = 86.76 \text{ lb} \downarrow}$




$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{136.19^2 + 86.76^2} = 161.47 \text{ lb}$$

$$\theta_R = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{86.76}{136.19}\right) = 32.5°$$

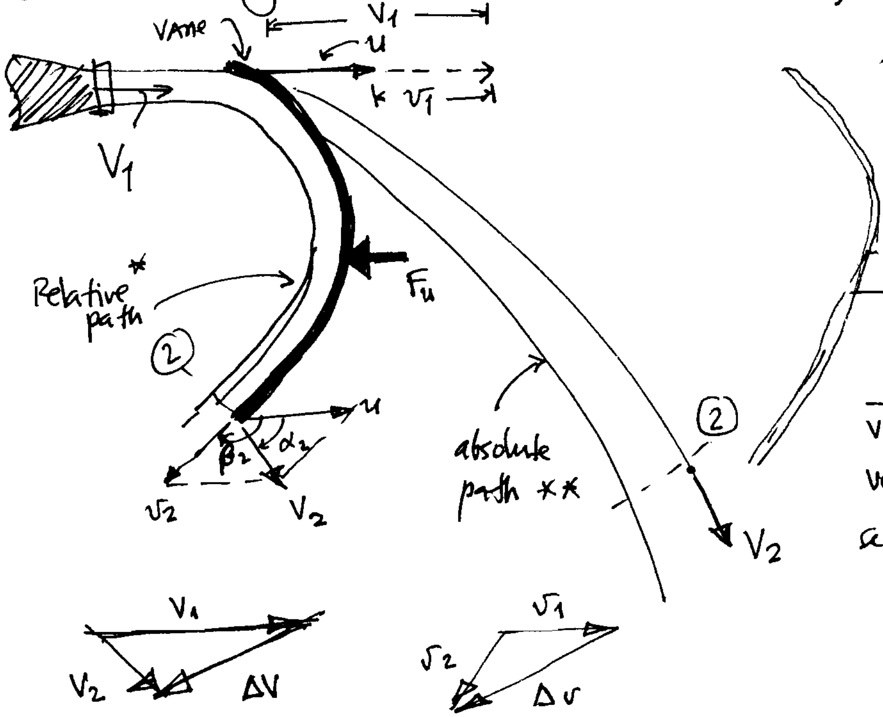
FORCE ON BLADE:



## 6.8. Force of a Jet on One or More Moving Vanes or Blades

Single blade, moving parallel to jet

jet leaves nozzle at (absolute) velocity, $V_1$




let $\vec{V}_1 = \vec{u} + \vec{v}_1$

also at section ②

$\vec{V}_2 = \vec{u} + \vec{v}_2$

$\vec{v}_1, \vec{v}_2$ = relative velocities at sections ① & ②

* path seen if moving with vane at speed $u \rightarrow$

** path seen by stationary observer

NOTE: if $u = V_1 \rightarrow$ FLUID can not reach the moving blade $\Rightarrow$ rate of fluid striking blade, $Q' = 0$.

If $u < V_1 \rightarrow$ rate of fluid striking blade, $Q' = A_1 v_1 = A_1 (V_1 - u)$

↑ relative velocity

mass rate, $\dot{m}' = \rho Q' = \rho A_1 v_1 = \rho A_1 (V_1 - u)$

$Q', \dot{m}'$ = relative discharge and mass rate for moving vane

momentum equation: $\sum \vec{F} = \dot{m}' \Delta \vec{V}$

$\Delta \vec{V} = \vec{V}_2 - \vec{V}_1 = (\vec{u} + \vec{v}_2) - (\vec{u} + \vec{v}_1) = \vec{v}_2 - \vec{v}_1 = \Delta \vec{v}$ ← change in velocities same whether using absolute or relative velocities

thus, $\boxed{\sum \vec{F} = \dot{m}' \Delta \vec{V} = \dot{m}' \Delta \vec{v} = \rho Q' \Delta \vec{V} = \rho Q' \Delta \vec{v}}$