Cavitation (continued)

Some locations that favor cavitation:

1- high point in a ~~[illegible]~~ siphon
2- contractions in a pipe (higher velocities $\Rightarrow$ lower pressure)
3- suction line in a pump

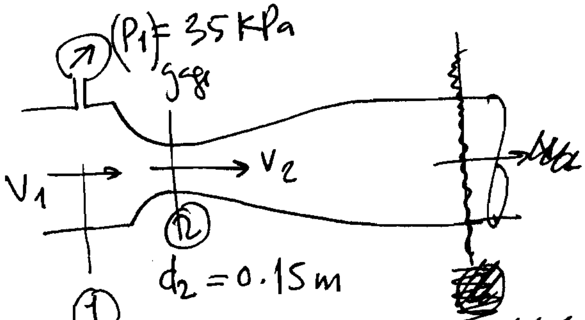
EXERCISES

5.10.2. Water at 40°C flows horizontally through a constriction similar to that in Fig. S5.9 when P_{atm} = 715 mmHg. The gage reading is 35 kPa, $d_1 = 0.5$ m, and $d_2 = 0.15$ m. Neglecting head loss, find the flow rate at which cavitation begins.

SOLUTION

$P_{atm} = 715 \text{ mmHg} = 95.33 \text{ kPa}$, $z_1 = z_2$

$h_L = 0$



$d_1 = 0.5$ m

$$\frac{P_1}{\gamma} + \cancel{z_1} + \frac{V_1^2}{2g} - \cancel{h_L}^{\,0} = \frac{P_2}{\gamma} + \cancel{z_2} + \frac{V_2^2}{2g}$$

$$V_1^2 - V_2^2 = \frac{\rho}{2}(P_2 - P_1)_{abs}$$

$(P_2)_{abs} = P_v = 7.38$ kPa for T=40°C (TABLE A.1, p.732)

$(P_1)_{abs} = (P_1)_{gage} + P_{atm} = 35 + 95.33$
$= 130.33$ kPa

$$V_1 = \frac{4Q}{\pi d_1^2}, \quad V_2 = \frac{4Q}{\pi d_2^2}$$

$$V_1^2 = \frac{16Q^2}{\pi^2 d_1^4}, \quad V_2^2 = \frac{16Q^2}{\pi^2 d_2^4}$$

$$V_1^2 - V_2^2 = \frac{16Q^2}{\pi^2}\left(\frac{1}{d_1^4} - \frac{1}{d_2^4}\right)$$

$$= \frac{16Q^2}{\pi^2}\left(\frac{1}{0.5^4} - \frac{1}{0.15^4}\right) =$$

$$= -3176.31\,Q^2$$

$$\frac{\rho}{2}(P_2 - P_1)_{abs} = \frac{992.2}{2}(7.38 - 130.33) \times 10^3$$

$$= -60995495.4$$

$$-3176.31\,Q^2 = -60995495.4$$

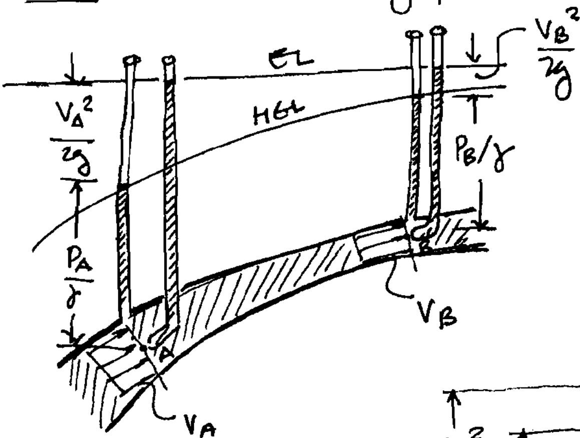
$$Q^2 = 19203.25$$

$$\boxed{Q = 138.57 \text{ m}^3/\text{s}}$$

NOTE: $\rho = 992.2$ kg/m³ for T=40°C (TABLE A.1, p.732)

5.11. Definition of Hydraulic Grade Line (HGL) and Energy Line (EL)

- piezometric head $= \frac{p}{\gamma} + z$, level in a piezometer tube
- HGL $\equiv$ piezometric line: line drawn through liquid surfaces in piezometers
- EGL $\Rrightarrow$ line indicated by pitot tubes

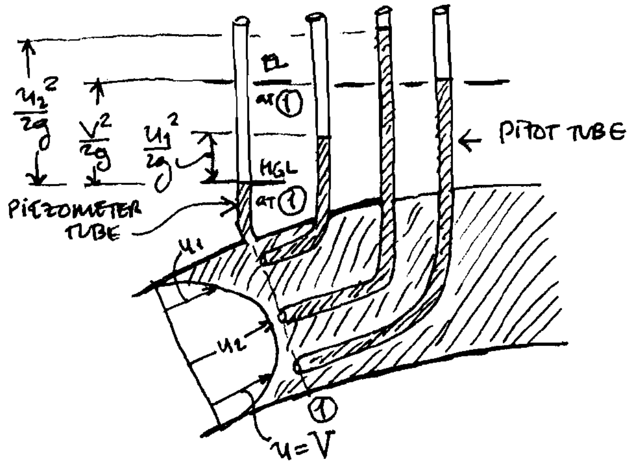


HORIZONTAL E.L. means NO ENERGY LOSSES

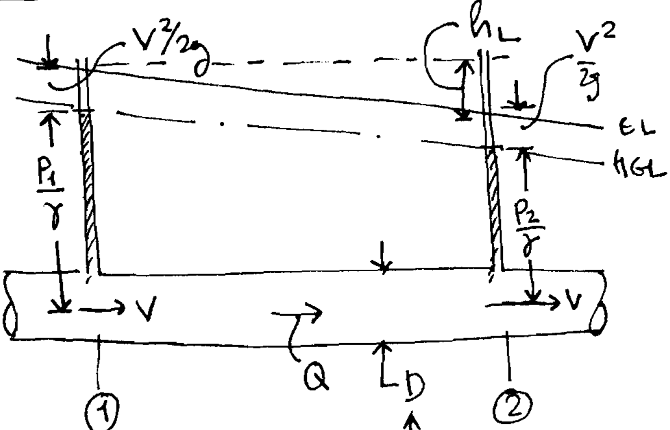
$\boxed{\text{IDEAL FLUID}}$

Pitot tubes measuring local velocities in a $\boxed{\text{REAL FLUID}}$

EL located where $u = V$ by pitot tube



EL & HGL in a horizontal pipeline



constant diameter

ENERGY EQN:

$$\frac{p_1}{\gamma} - h_L = \frac{p_2}{\gamma}$$

$Q =$ constant (steady flow)

$A = \frac{\pi D^2}{4} =$ constant

$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} =$ constant

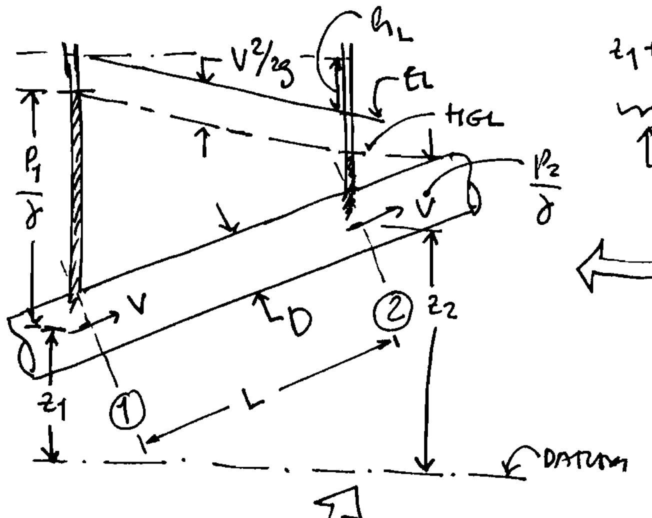
DISTANCE BETWEEN HGL & EL is constant $= \frac{V^2}{2g}$ (the velocity head)

$h_L = \frac{\tau_0 P L}{\gamma A} \sim L$, since $\tau_0, P, \gamma, A =$ constant

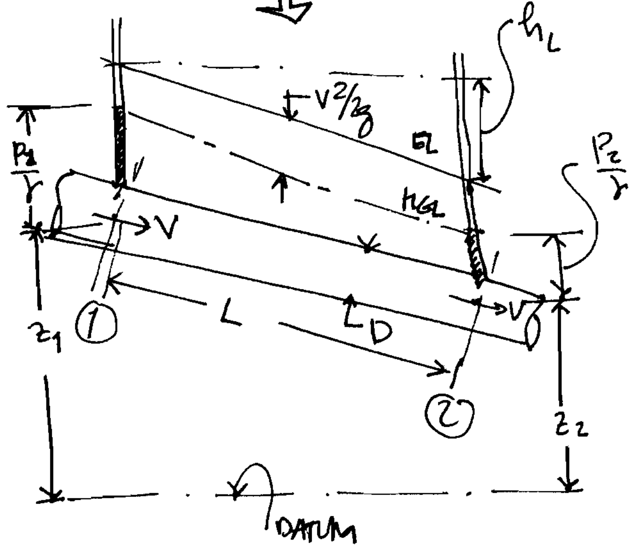
HGL & EL in inclined pipelines (REAL FLUIDS)

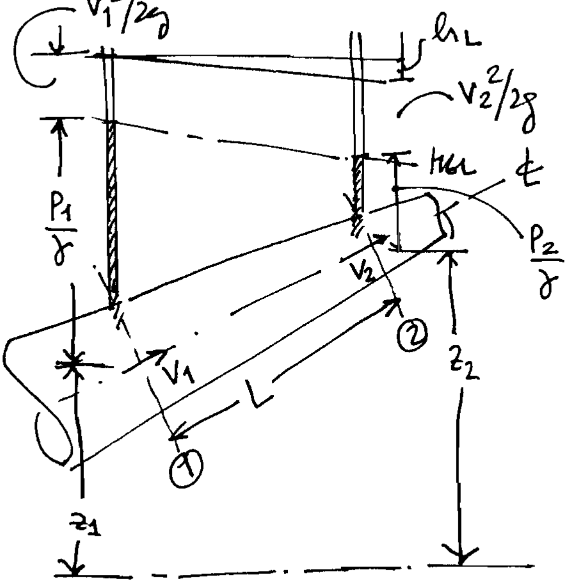
ENERGY EQUATION

$$\underbrace{z_1+\frac{p_1}{\gamma}}_{\text{piezometric heads}}-h_L=\underbrace{z_2+\frac{p_2}{\gamma}}_{\text{piezometric heads}}$$



constant diameter





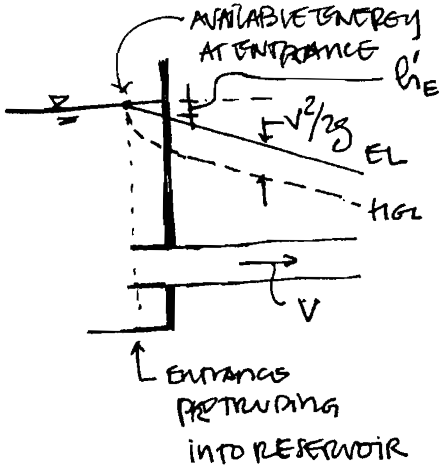
Q = constant

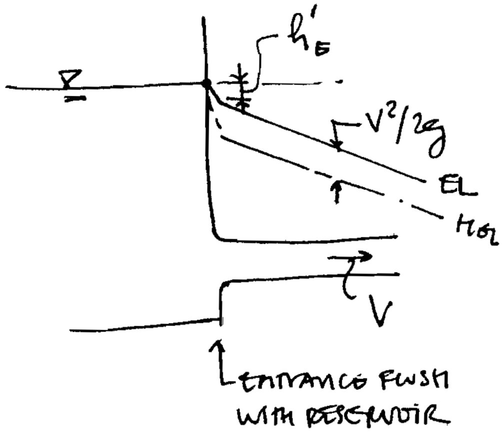
$$V_2>V_1 \Rightarrow \frac{V_2^2}{2g}>\frac{V_1^2}{2g}$$

ENERGY EQUATION

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

HGL & EL for flow from reservoir to pipe



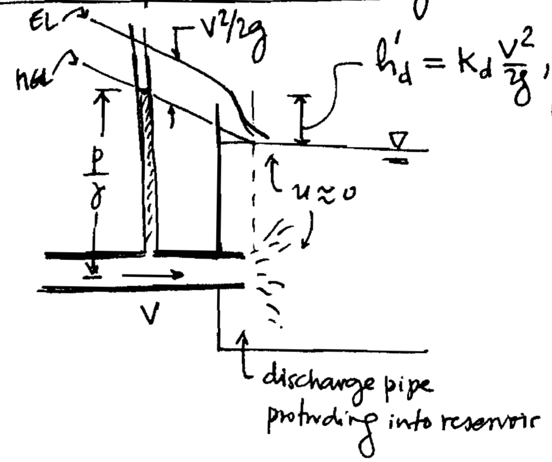


Some energy is lost due to liquid "negotiating" the entrance into the pipeline. This is an example of a minor loss (or local loss). The entrance loss is written as

$$h'_E = K_E \frac{V^2}{2g}$$

K_E = entrance loss coefficient (SEE SECTION 8.21, pp. 302-303)

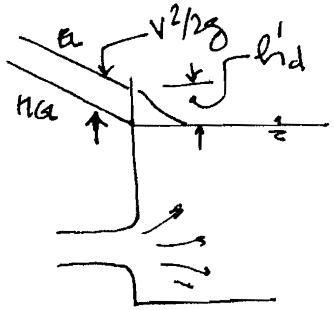
5.12. loss of head at submerged discharge

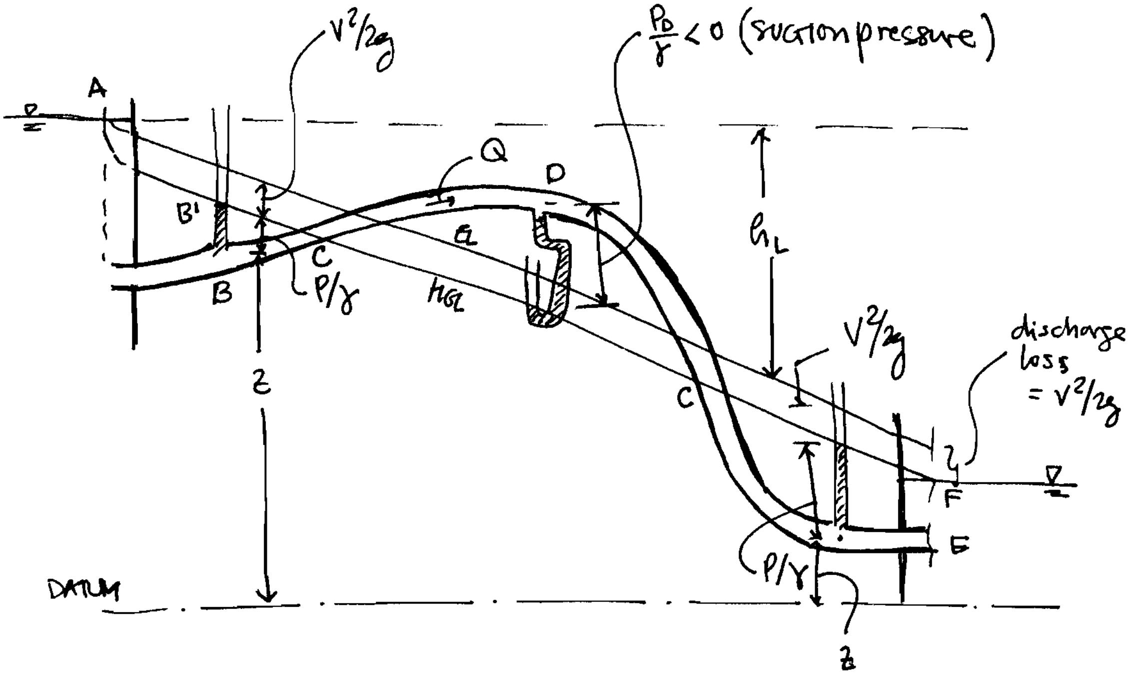


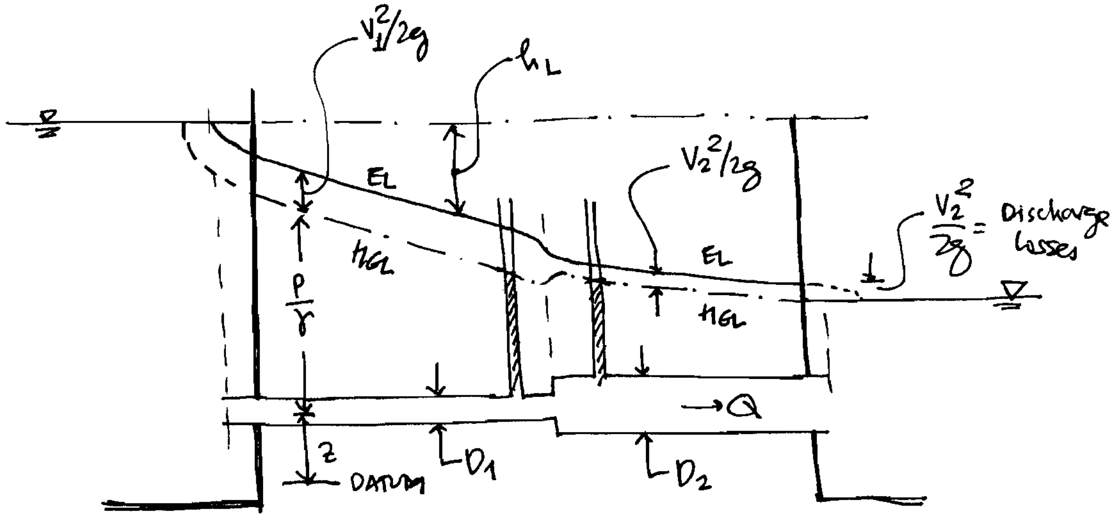
$h'_d = K_d \frac{V^2}{2g}$, with $K_d = 1.0$

minor or local losses

i.e., the loss at a submerged discharge is equal to the velocity head at the pipe



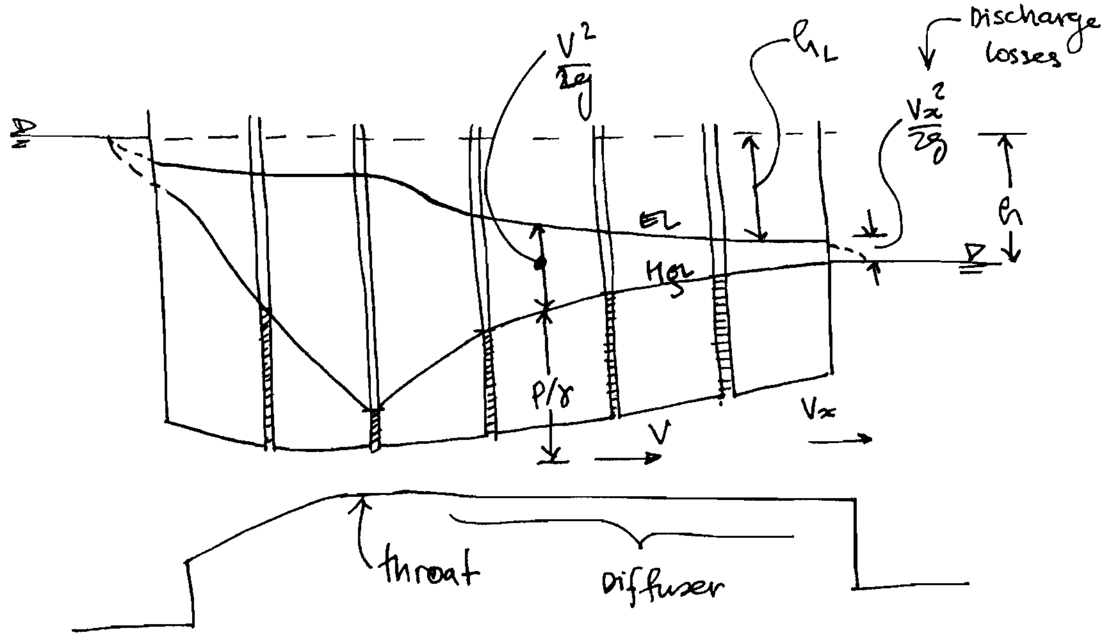




$Q = \text{constant}$, with $D_2 > D_1 \Rightarrow V_1 > V_2$

$$V_1 = \frac{4Q}{\pi D_1^2}$$

$$V_2 = \frac{4Q}{\pi D_2^2}$$



VENTURI METER

$D_1 > D_2$, $V_1 < V_2$

also $P_1 > P_2$, use

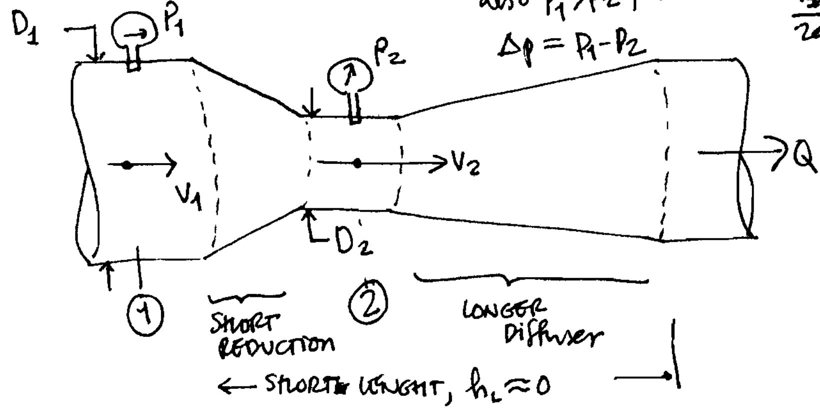
$\Delta p = P_1 - P_2$

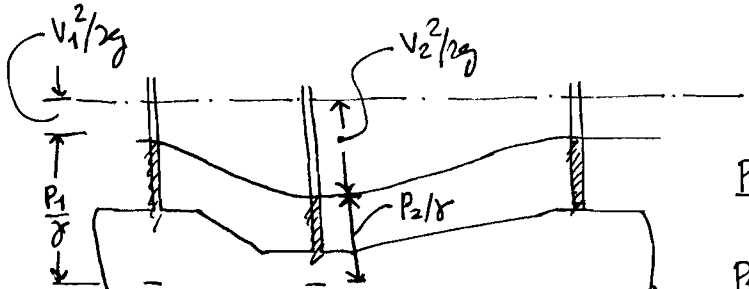
$$V_1 = \frac{4Q}{\pi D_1^2}$$

$$\frac{V_1^2}{2g} = \frac{8Q^2}{\pi^2 g D_1^4}$$

$$V_2 = \frac{4Q}{\pi D_2^2}$$

$$\frac{V_2^2}{2g} = \frac{8Q^2}{\pi^2 g D_2^4}$$





ENERGY EQUATION

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

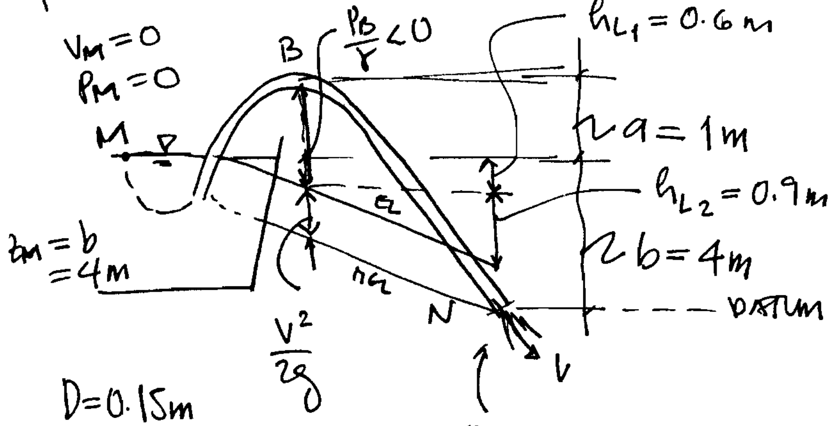
$$\frac{P_1 - P_2}{\gamma} = \frac{8Q^2}{\pi^2 g}\left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right)$$

$$Q^2 = \frac{\pi^2}{8\left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right)} \frac{\Delta p}{\rho} \Rightarrow Q = \sqrt{\frac{\pi^2}{8\left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right)} \frac{\Delta p}{\rho}}$$

$$Q \sim \sqrt{\Delta p}$$

EXERCISES

5.13.1. Assume there is friction head loss in the siphon of Fig. X5.13.1, where a = 1m, b = 4m. The loss between the intake and B is 0.6 m and between B and N is 0.9m. What is the rate of discharge and pressure head at B when the diameter is 150 mm?



$P_B = ?$
$z_B = a + b = 5$m
$V_B = V$

$\gamma = 9810\ N/m^3$
$\gamma = 9.81\ kN/m^3$

D = 0.15m

$P_N = 0$
$V_N = V = \frac{4Q}{\pi D^2}$

ENERGY
M-N

$$\cancel{\frac{P_M}{\gamma}}^{0} + z_M + \cancel{\frac{V_M^2}{2g}}^{0} - (h_{L_1} + h_{L_2}) = \cancel{\frac{P_N}{\gamma}}^{0} + \cancel{z_N}^{0} + \frac{V_N^2}{2g}$$

$$0 + 4 + 0 - (0.6 + 0.9) = \frac{V_N^2}{2 \times 9.81}$$

$$V_N^2 = V^2 = 2 \times 9.81 \times (4 - 1.5) = 49.05$$

$$V_N = V = \sqrt{49.05} = 7.00\ m/s$$

$$Q = V \cdot \frac{\pi D^2}{4} = (7.00) \times \frac{\pi \times 0.15^2}{4} = 0.1237\ m^3/s$$

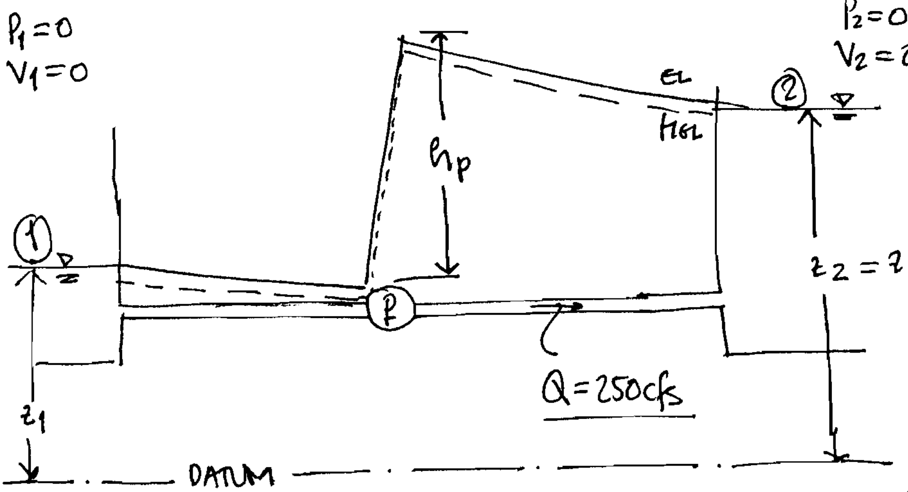
or $Q = 123.7\ L/s$

ENERGY M-B: $\frac{P_M}{\gamma} + z_M + \frac{V_M^2}{2g} - h_{L_1} = \frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g}$

$$0 + 4 + 0 - 0.6 = \frac{P_B}{\gamma} + 5 + \frac{7^2}{2 \times 9.81}$$

$$3.4 = \frac{P_B}{\gamma} + 7.497 \Rightarrow P_B = -4.09\,m \times 9.81\ \frac{kN}{m^3} = -40.19\ kPa$$

5.13.5. A pump, having an efficiency of 90%, lifts water to a height of 465 ft at a rate of 250 cfs. The friction head loss in the pipe is 35 ft. What is the required horsepower? Also sketch the energy line and the hydraulic grade line of this system.



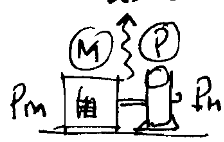
ENERGY EQUATION: $\left(\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) - h_L + h_p = \left(\frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right)$

$$0 + \cancel{z_1} + 0 - 35 + h_p = 0 + \cancel{z_1} + 465 + 0$$

$$\underline{h_p = 465 + 35 = 500 \text{ ft}}$$

Pump hydraulic power: $P_H = \frac{\gamma Q h_p}{550} = \frac{62.4 \times 250 \times 500}{550}$ $h_p = 14181.81$ hp

Deffinition of efficiency: $\eta = \frac{P_H}{P_M}$ ← hydraulic power / ← motor power



$\frac{P_H}{P_M} = \frac{\text{OUTPUT}}{\text{INPUT}} < 1$

$$P_M = \frac{P_H}{\eta} = \frac{14181.81 \text{ hp}}{0.90} = 15757.57 \text{ hp}$$