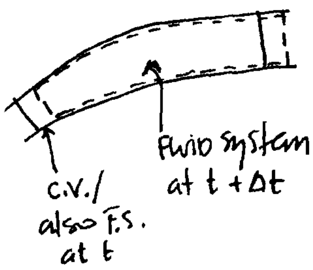
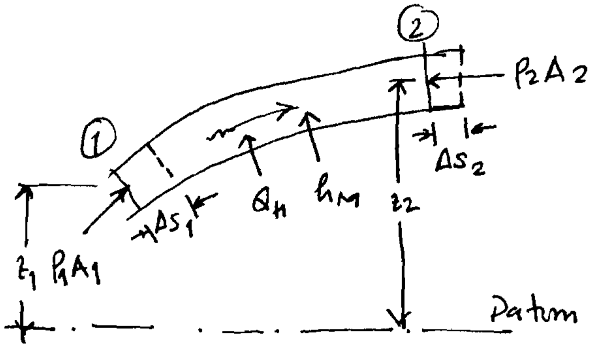
5.5. General energy equation for steady flow of any fluid





steady flow $\Rightarrow$ continuity: $\underbrace{\gamma_1 A_1 \Delta s_1}_{} = \underbrace{\gamma_2 A_2 \Delta s_2}_{} = g\Delta m$

$\gamma_1 \forall_1 = \gamma_2 \forall_2 = g \cdot \Delta m$

Reynolds Transport Theorem with $X = E$ (energy): $\Delta E_S = \Delta E_{CV} + \Delta E_{CV}^{out} - \Delta E_{CV}^{in}$

steady state $\Rightarrow \Delta E_{CV} = 0 \Rightarrow \Delta E_S = \Delta E_{CV}^{out} - \Delta E_{CV}^{in}$

First law of thermodynamics: $\begin{pmatrix}\text{EXTERNAL} \\ \text{WORK} \\ \text{DONE}\end{pmatrix} + \begin{pmatrix}\text{HEAT} \\ \text{TRANSFERRED}\end{pmatrix} = \Delta E_S$

WORK

- Flow work: force moves mass a certain distance

$$\text{Flow work} = P_1 A_1 \Delta s_1 - P_2 A_2 \Delta s_2 = \frac{P_1}{\gamma_1}(\underbrace{\gamma_1 A_1 \Delta s_1}_{g\Delta m}) - \frac{P_2}{\gamma_2}(\underbrace{\gamma_2 A_2 \Delta s_2}_{g\Delta m})$$

$$= \left(\frac{P_1}{\gamma_1} - \frac{P_2}{\gamma_2}\right) \cdot g \cdot \Delta m$$

- Shaft work: work added or subtracted by a machine

$$\text{Shaft work} = \frac{\text{WEIGHT}}{\text{TIME}} \times \frac{\text{ENERGY}}{\text{WEIGHT}} \times \text{time} = \left(\gamma_1 A_1 \frac{\Delta s_1}{\cancel{\Delta t}}\right) h_M \cancel{\Delta t} = \underbrace{\gamma_1 A_1 \Delta s_1}_{g \cdot \Delta m} h_M$$

$$= g \cdot \Delta m \cdot h_M$$

pump: adds energy, $h_M > 0$

Turbine: removes energy, $h_M < 0$

heat transferred $= \left(\gamma_1 A_1 \frac{\Delta s}{\cancel{\Delta t}}\right) Q_H \cancel{\Delta t} = g \Delta m Q_H$

heat transferred per unit weight

(I) $\Delta E_S = \left(\frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2} + h_M + Q_H\right) g \Delta m$

TOTAL ENERGY = KINETIC + POTENTIAL + INTERNAL

$$\Delta E = g \Delta m \left(z + \alpha \frac{V^2}{2g} + I\right)$$

(II) Thus, $\Delta E_{cv}^{out} - \Delta E_{cv}^{in} = g \Delta m \left(z_2 + \alpha \frac{V_2^2}{2g} + I_2\right) - g \Delta m \left(z_1 + \alpha \frac{V_1^2}{2g} + I_1\right)$

(I) & (II) into RTT for energy (divide by $g \cdot \Delta m$):

$$\frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2} + h_M + Q_H = \left(z_2 + \alpha \frac{V_2^2}{2g} + I_2\right) - \left(z_1 + \alpha \frac{V_1^2}{2g} + I_1\right)$$

$$\boxed{\left(\frac{p_1}{\gamma_1} + z_1 + \alpha \frac{V_1^2}{2g} + I_1\right) + h_M + Q_H = \left(\frac{p_2}{\gamma_1} + z_2 + \alpha \frac{V_2^2}{2g} + I_2\right)}$$

only assumption: STEADY STATE

5.6. Energy equations for steady flow of incompressible fluids

γ = constant, $\alpha = 1$ (ok for turbulent flow, small error in laminar)

$$\left(\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) + h_M + Q_H = \left(\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right) + \underbrace{(I_2 - I_1)}_{}$$

machines — HEAT

$\frac{\Delta(\text{internal energy})}{W}$

Δ(int. energy)

$$\frac{\cancel{\Delta E}}{m} = \Delta i = i_2 - i_1 = c \Delta T = c(T_2 - T_1), \quad c = \text{heat capacity}$$

$$\frac{\Delta(\text{int. energy})}{W} = \Delta I = \frac{\Delta i}{g} = I_2 - I_1 = \frac{c}{g}\Delta T = \frac{c}{g}(T_2 - T_1) = Q_H + h_L$$

head loss

$$\boxed{\left(\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) + h_M - h_L = \left(\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right)}$$

General steady flow equation (energy) with $\alpha = 1.0$ [ok for most turbulent flow]:

$$\underbrace{\left(\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right)}_{\substack{\text{TOTAL HEAD}\\ \text{AT SECTION } 1\\ \text{[upstream]}}} + \underbrace{h_M}_{\substack{\text{machine}\\ \text{energy input}}} - \underbrace{h_L}_{\substack{\text{HEAD}\\ \text{LOSSES}}} = \underbrace{\left(\frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right)}_{\substack{\text{TOTAL HEAD}\\ \text{AT SECTION } 2\\ \text{[DOWNSTREAM]}}}$$

$\Rightarrow$ NOTE: $\boxed{h_M = \underbrace{h_P}_{\text{pump}} - \underbrace{h_T}_{\text{TURBINE}}}$

- If no machine present, $h_M = 0$, ENERGY EQUATION becomes

$$\left(\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) - h_L = \left(\frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right)$$

- ENERGY losses: $h_L = \underbrace{h_f}_{\text{FRICTION losses}} + \underbrace{h_m}_{\text{minor losses}}$

minor losses: VALVES, ELBOWS, NOZZLES, etc. } in pipelines

FRICTION losses $\rightarrow$ VISCOSITY & TURBULENCE

- in IDEAL FLUIDS or if losses are negligible, $h_L = 0$, and the energy equation reduces to Bernoulli's theorem

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

or,

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = \text{CONSTANT}$$

EXERCISES

5.6.1. Water is flowing in a pipeline. Due to heat input from the enviroment and energy dissipation (head loss), the water temperature rises by 3°F between intake and outlet. Find the gain in heat in (a) ft·lb/lb, (b) Btu/lb

NOTE: In Table A.4 ⟹ Fresh water's specific heat

$$c = 25000\ \frac{ft\cdot lb}{slug\cdot {}^{o}R} = 25000\ \frac{ft^2}{sec^2\ {}^{o}R}$$

In this problem $\Delta T = T_2 - T_1 = 3^{o}F \equiv 3^{o}R$

$$\Delta i = c\cdot\Delta T = 25000\ \frac{ft\cdot lb}{slug\cdot\cancel{{}^{o}R}} \times 3\cancel{{}^{o}R} = 75000\ \frac{ft\cdot lb}{slug}$$

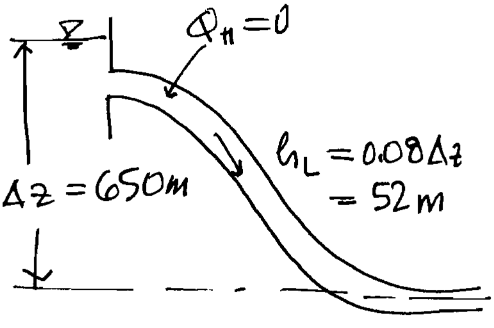
In Eq. (5.25) $\quad \dfrac{\Delta(\text{internal energy})}{\text{unit weight}} = \dfrac{c}{g}\Delta t = \underbrace{Q_H + h_L}_{\text{change in heat } \Delta Q} = \Delta Q$

$$\Rightarrow \Delta Q = \frac{\Delta i}{g} = \frac{75000\ ft\cdot lb/slug}{32.2\ ft/s^2} = 2329.19\ \frac{ft\cdot lb}{lb}$$

Conversion factor $1\ ft\cdot lb = 1.285\times10^{-3}\ Btu$

$$\Delta Q = 2329.19 \times \frac{1.285\times10^{-3}\ Btu}{lb} = 2.99\ \frac{Btu}{lb}$$

5.6.4. The pipeline shown in Fig. X5.6.3 supplies water to a hydroelectric power plant, the elevation of which is 650 m below the level of the water surface at intake to the pipe. If 8% of this total, or 52 m, is the head loss in the line, what will be the value of ΔI in J/N if there is no heat transfer, and what will be the rise in temperature?



Eq. (5.25): $\Delta I = Q_H + h_L$

$\Delta I = 0 + 52^m$

NOTE: $1J = 1N \times m$

$\Rightarrow 1\frac{J}{N} = 1m$

Thus, $\boxed{\Delta I = 52\frac{J}{N}}$

To calculate the raise in temperature $\Delta T = ?$ use

$$\Delta I = \frac{c}{g}\Delta T \Rightarrow \Delta T = \frac{g\Delta I}{c} = \frac{(9.81\,m/s^2)(52\,m)}{4187\,m^2/s^2K}$$

$c = 4187\,\frac{m^2}{s^2 \cdot K}$

$g = 9.81\,m/s^2$

$$\boxed{\Delta T = 0.12K = 0.12\,°C}$$

5.7. Energy equation for steady flow of compressible fluids

- ENERGY EQUATION (5.31) with $h_M = 0$, $P/\gamma \gg z_2 - z_1$, also introduce enthalpy $\Rightarrow$ $h = i + \frac{p}{\rho} = gI + \frac{p}{\rho} \Rightarrow I + \frac{p}{\gamma} = \frac{h}{g}$

$$\boxed{\frac{h_1}{g} + \frac{V_1^2}{2g} + Q_H = \frac{h_2}{g} + \frac{V_2^2}{2g}}$$ ← The h is ENTHALPY

(see Sample Problem 5.6, Exercises pp. 149–150)

5.8. Head

$\frac{p}{\gamma}$ = pressure head, z = elevation head or potential head

$\frac{V^2}{2g}$ = velocity head $\Rightarrow$ $\boxed{H = \underbrace{\frac{p}{\gamma} + z}_{\text{piezometric head or static (pressure) head}} + \frac{V^2}{2g}}$ $\leftarrow$ TOTAL HEAD

- Energy equations:

1) ideal fluid ($h_L = 0$) with no machines ($h_m = 0$): $H_1 = H_2$
2) real fluids ($h_L \neq 0$) with no machines ($h_m = 0$): $H_1 - h_L = H_2$
3) real fluids ($h_L \neq 0$) with machines ($h_m \neq 0$): $H_1 - h_L + h_m = H_2$

- Machine heads $\begin{cases} h_m = h_p & \text{for a pump} \\ h_m = -h_t & \text{for a turbine} \end{cases}$

5.9. Power considerations in fluid flow

F = force, V = velocity
T = torque, ω = angular velocity

Definition: $P = \frac{\text{WORK}}{\text{TIME}} = \frac{\text{FORCE} \times \text{LENGTH}}{\text{TIME}} = F \cdot V = T \cdot \omega$

$F \cdot V = (\Delta p \cdot A) \cdot V = \gamma h \cdot A \cdot V = \gamma h Q \Rightarrow \boxed{P = Q \Delta p = \gamma Q h}$

γ = specific weight [lb/ft³, N/m³], Q = discharge [ft³/s, m³/s]
h = energy head [ft, m] , p = pressure [lb/ft², Pa]
P = power [ft·lb/sec, W]

Units more commonly used:

BG units: Horsepower $= P = \frac{\gamma Q h}{550} = \frac{Q \Delta p}{550}$

SI units: Kilowatts $= P = \frac{\gamma Q h}{1000} = \frac{Q \Delta p}{1000}$

The head h can be any head (or head difference) and Δp any pressure difference

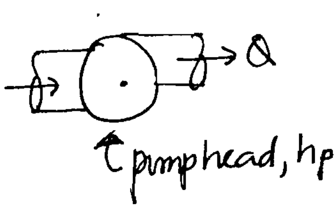
- For a turbine, $h = h_t$
- For a pump, $h = h_p$
- For a jet flowing at velocity V_j, $h = V_j^2/2g$
- For friction losses, $h = h_f$

etc.

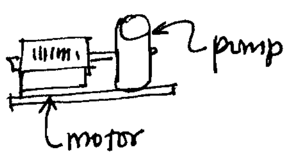
When power is transmitted through a process or machine, ~~define~~ define EFFICIENCY η as:

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

- e.g., in a pump, $P_H = \gamma h_p Q$ = power introduced into the flow (OUTPUT)

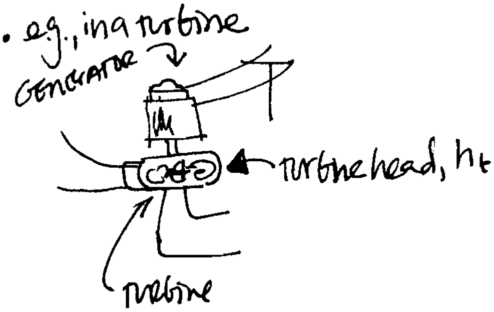


P_M = power used by motor (INPUT)



$$\eta = \frac{P_H}{P_M} < 1$$

- e.g., in a turbine



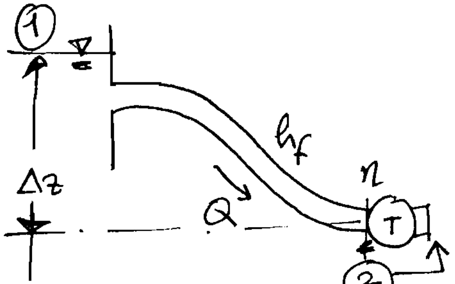
$P_H = \gamma h_t Q$ = power extracted by turbine from flow [INPUT]

P_E = power produced by the generator (OUTPUT)

$$\eta = \frac{P_E}{P_H} < 1$$

EXERCISES.

5.9.2. A turbine, located 255 m below the water surface at intake (Fig. X5.9.1), carries a flow of 3.5 m³/s. The head loss in the pipeline leading to it is 10 m. Find the power (kW) delivered by the turbine if its efficiency is 92%.



SOLUTION

$Q = 3.5\,m^3/s$, $h_L = 10\,m$

$P_1 = 0$, $V_1 = 0$, $z_1 = \Delta z = 255\,m$

$P_2 = 0$, $V_2 = \cancel{\frac{V_2^2}{2g}} \approx 0$, $z_2 = 0$

$$\cancel{\frac{P_1}{\gamma}}^{0} + z_1 + \cancel{\frac{V_1^2}{2g}}^{0} - h_t - h_L = \cancel{\frac{P_2}{\gamma}}^{0} + z_2 + \cancel{\frac{V_2^2}{2g}}^{0}$$

$$0 + 255 + 0 - h_t - 10 = 0 + 0 + 0 \Rightarrow h_t = 245\,m$$

$$P_H = \gamma h_t Q$$

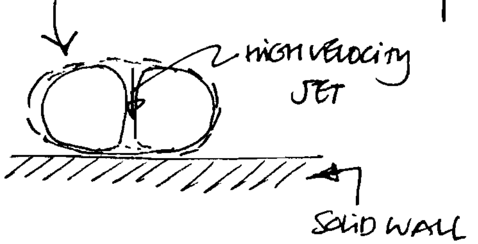
$$P_E = \eta P_H = \eta \gamma h_t Q = 0.92 \times 9810 \frac{N}{m^3} \times 245\,m \times 3.5\,m^3/s$$

$$P_E = 7{,}739{,}109 \frac{N \cdot m}{s} = 7739\,kW$$

$$J/s$$

$$W$$

5.10. Cavitation · zones of low pressure, if $P_{abs} = P_v$ (vapor pressure) ⇒ vapor cavities. When swept into points of larger pressure, the cavities implode – resulting jet has $V \approx 360$ ft/s (110 m/s) and pressures, $p \approx 500$ atm (7350 psi, 50.7 MPa)



Imploding cavities damage turbine runners, pump impellers, ship propellers, etc.

To avoid cavitation, keep $P_{abs} > P_v$