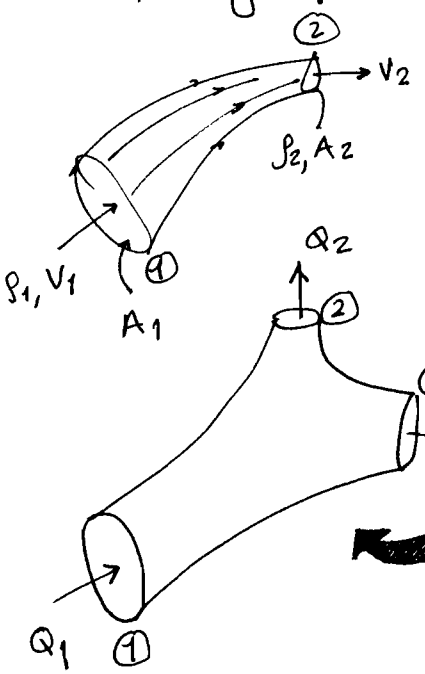
Continuity equation in branching conduits

The continuity equation for a single conduit is given by any of the following expressions (steady flow):

- mass flow: $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$
- weight flow: $G = g \cdot \dot{m} = \gamma_1 A_1 V_1 = \gamma_2 A_2 V_2$
- volume flow: $Q = A_1 V_1 = A_2 V_2$ (liquids)

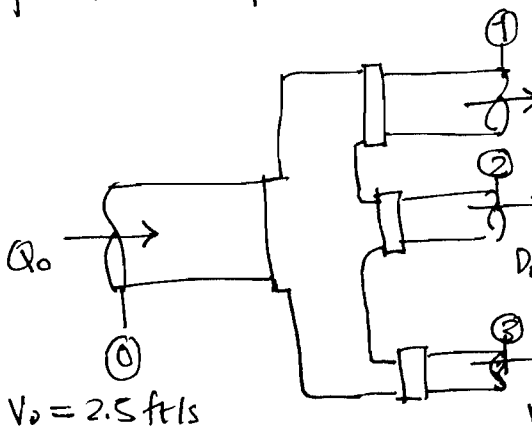


If the conduit branches into 2 or more conduits, then the continuity equation is modified accordingly, e.g.,

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3, \quad G_1 = G_2 + G_3, \text{ or}$$

$$Q_1 = Q_2 + Q_3$$

EXAMPLE: a pipeline 3 ft in diameter branches through a manifold into 3 pipes of diameters 1.0, 1.5, and 2.0 ft, respectively. If water flows through the main pipeline at a velocity of 2.5 ft/s, and if the discharge is equally split among the 3 branching pipelines, what is the velocity of the flow in each of the branches?



continuity:

$$Q_0 = Q_1 + Q_2 + Q_3$$

also, $Q_1 = Q_2 = Q_3 = Q$ (given) $\Rightarrow Q_0 = 3Q$

$$Q = \frac{Q_0}{3} = \frac{1}{3} V_0 A_0 = \frac{1}{3} V_0 \frac{\pi D_0^2}{4}$$

$$Q = \frac{\pi}{12} V_0 D_0^2 = \frac{\pi}{12} \times 2.5 \times 3^2$$

$$Q = 5.89 \text{ cfs} = Q_1 = Q_2 = Q_3$$

$$Q_i = V_i A_i = V_i \cdot \frac{\pi D_i^2}{4} \Rightarrow V_i = \frac{4 Q_i}{\pi D_i^2} \Rightarrow V_1 = \frac{4 \times 5.89}{\pi \times 1^2} = 7.5 \text{ fps}$$

$$V_2 = \frac{4 \times 5.89}{\pi \times 1.5^2} = 3.33 \text{ fps}, \quad V_3 = \frac{4 \times 5.89}{\pi \times 2^2} = 1.87 \text{ fps}$$