The most efficient trapezoidal cross-section for uniform flow is half of a regular hexagon, as illustrated in the figure below. What is the bottom width $b$ and the depth of flow $y$ for this cross-section if a concrete-line channel ($n = 0.012$) is laid on a slope of 0.00025 and carries a discharge of 20 cfs?

The figure above shows that the side slope $m$ is $m = \tan(30^\circ)$, i.e.,

$$m := \tan\left(\frac{30 \cdot \pi}{180}\right)$$

$$= \frac{1}{3} \sqrt{3}$$

(1)

The depth of flow is

$$y := b \cdot \cos\left(\frac{30 \cdot \pi}{180}\right)$$

$$= \frac{1}{2} b \sqrt{3}$$

(2)

The area, wetted perimeter and hydraulic radius are:

$$A := (b + m \cdot y) \cdot y$$

$$= \frac{3}{4} b^2 \sqrt{3}$$

(3)

$$P := b + 2 \cdot y \cdot \sqrt{1 + m^2}$$

$$= 3 b$$

(4)

$$Rh := \frac{A}{P}$$

$$= \frac{1}{4} b \sqrt{3}$$

(5)

Manning's equation is:

$$Eq\text{Manning} := Q = \frac{1.486}{n} \cdot A \cdot Rh^\frac{2}{3} \cdot \sqrt{So}$$

$$Q = 0.278625000 b^2 \sqrt{3} \left(\frac{3}{4} \right)^{1/3} \left(\frac{b}{\sqrt{3}}\right)^{2/3} \sqrt{So}$$

(6)

For

$$Q := 20 ; n := 0.012 ; So := 0.00025 ;$$

Manning's equation becomes:
\[ 20 = 0.3671206722 \ b^2 \sqrt{3} \ 4^{1/3} \left( b \sqrt{3} \right)^{2/3} \]  

(7)

Solving for \( b \):

\[ \text{solve}(\text{EqManning}, \ b) \]

\[-1.888904688 + 1.888904688 \ I, -1.888904688 - 1.888904688 \ I, 2.671314628 \]

(8)

The real solution is:

\[ b := 2.67 \]  

(9)

While the depth is:

\[ \text{evalf}(y) \]

\[ 2.312287829 \]  

(10)


A trapezoidal channel with side slopes of 1 on 1 carries a flow of 20.4 m\(^3\)/s. For a depth of 4.88 m, calculate the critical velocity.

The equation for critical conditions is:

\[ \text{restart} : \text{EqCritical} := \frac{Q^2 \cdot B}{g \cdot A^3} = 1 \]

\[ \frac{Q^2 B}{g A^3} = 1 \]  

(11)

\[ Q := 20.4 \quad b := 4.88 \quad m := 1 \quad g := 9.806 \quad B := b + 2 \cdot m \cdot y \]

\[ 4.88 + 2 \cdot y \]  

(12)

\[ A := \left( b + m \cdot y \right) \cdot y \]

\[ (4.88 + y) \cdot y \]  

(13)

Then, the equation for critical conditions is:

\[ \text{EqCritical} \]

\[ \frac{42.43932286 \left( 4.88 + 2 \cdot y \right)}{(4.88 + y)^3} y^3 = 1 \]  

(14)

The critical depth is calculated as follows:

\[ \text{solve}(\text{EqCritical}, \ y) \]

\[ 1.118523379, -0.5514019925 + 1.153416716 \ I, -5.529180118 + 0.9591405229 \ I, -3.597359157, -5.529180118 - 0.9591405229 \ I, -0.5514019925 - 1.153416716 \ I \]

(15)

Use:

\[ y := 1.12 \]  

(16)

Then,

\[ A \]

\[ 6.7200 \]  

(17)

and, the velocity is:

\[ V := \frac{Q}{A} \]

\[ 3.035714286 \]  

(18)
A rectangular channel 12.0 m wide is laid on a slope of 0.0028. The depth of flow at one section is 1.80 m while the depth of flow at another section 500 m downstream is 1.5 m. Determine the flow if $n = 0.026$.

**restart**: $b := 12 : So := 0.0028 : y1 := 1.5 : Dx := 500 : y2 := 1.8 : n := 0.026 : g := 9.806$

First, we calculate the area, wetted perimeter, and hydraulic radius for sections 1 and 2, and an average hydraulic radius, $Rhmean$:

\[
\begin{align*}
\text{AI} & := b \cdot y1 \\
P1 & := b + 2 \cdot y1 \\
Rh1 & := \frac{\text{AI}}{P1} \\
A2 & := b \cdot y2 \\
P2 & := b + 2 \cdot y2 \\
Rh2 & := \frac{A2}{P2} \\
Rhmean & := \frac{1}{2} \cdot (Rh1 + Rh2) \\
\end{align*}
\]

\begin{align*}
18.0 \\
15.0 \\
1.200000000 \\
21.6 \\
15.6 \\
1.384615385 \\
1.292307692
\end{align*}

The velocities of the two sections is calculated in terms of the discharge. Also, a mean velocity is calculated as $Vmean$:

\[
\begin{align*}
V1 & := \frac{Q}{A1} \\
V2 & := \frac{Q}{A2} \\
Vmean & := \frac{1}{2} \cdot (V1 + V2) \\
\end{align*}
\]

\begin{align*}
0.0555555556 \ Q \\
0.04629629630 \ Q \\
0.05092592593 \ Q
\end{align*}

From the Manning's equation we obtain an expression for the mean energy slope, still as a function of the discharge $Q$:

\[
\begin{align*}
EqManning & := Vmean = \frac{1}{n} \cdot Rhmean \ \frac{2}{3} \cdot \sqrt{S} \\
0.05092592593 \ Q & = 45.63211569 \ \sqrt{S} \\
Smean & := \text{solve}(EqManning, S) \\
0.000001245479714 \ Q^2
\end{align*}
\]

Next, we calculate the specific energies at sections 1 and 2, still as functions of the discharge:
\[ E1 := y_1 + \frac{V_1^2}{2 \cdot g} \quad 1.5 + 0.0001573740442 \, Q^2 \]  
\[ E2 := y_2 + \frac{V_2^2}{2 \cdot g} \quad 1.8 + 0.0001092875306 \, Q^2 \]  

The equation of gradually varied flow is:

\[ EqGVF := D_x = \frac{E_1 - E_2}{S_{mean} - S_0} \]

\[ 500 = \frac{-0.3 + 0.0000480865136 \, Q^2}{0.000001245479714 \, Q^2 - 0.0028} \]  

\[ \text{solve}(EqGVF, Q) \quad 43.75154296, -43.75154296 \]

The solution is, therefore, \( Q = 43.75 \, m^3/s \)

[4] An 8-ft-wide rectangular channel contains a sluice gate which extends across the full width of the channel with an opening of 0.55 ft. Assuming \( C_d = 0.60 \), \( C_c = 0.60 \), and free flow, find the flow rate and the sluice coefficient when the upstream depth is 4.8 ft.

\[ \text{restart} : b := 8 \quad : a := 0.55 \quad : C_d := 0.60 \quad : C_c := 0.60 \quad : y_1 := 4.8 \quad : g := 32.2 \quad : \]

The flow downstream in free flow is calculated as:

\[ y_2 := C_c \cdot a \quad 0.3300 \]  

The ideal velocity at section 2 is given by the following equation:

\[ V_{2i} := \frac{1}{\sqrt{1 - \frac{A_2}{A_1}}} \cdot \sqrt{2 \cdot g \cdot (y_1 - y_2)} \quad \sqrt{1 - \frac{A_2}{A_1}} \]

\[ 16.96667322 \]

With

\[ A_1 := b \cdot y_1 \quad 38.4 \]  
\[ A_2 := b \cdot y_2 \quad 2.6400 \]  

the ideal velocity becomes:

\[ V_{2i} \quad 17.58180878 \]  

The discharge is calculated as follows:

\[ A := b \cdot a \quad 4.40 \]

\[ Q := C_d \cdot V_{2i} \cdot A \quad 46.41597519 \]

With this value, the sluice coefficient is calculated as:
\[ K_s := \frac{Q}{A \cdot \sqrt{2 \cdot g \cdot y l}} \]

0.6000000001

(42)