Notes on forces on curved surfaces, buoyancy and flotation

prepared by
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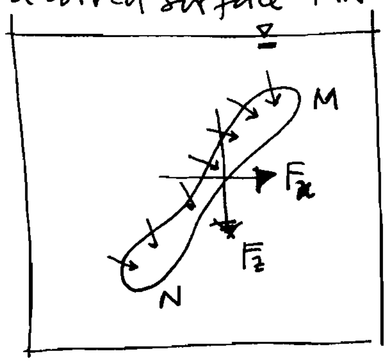
for
CEE 3500 – CEE Fluid Mechanics

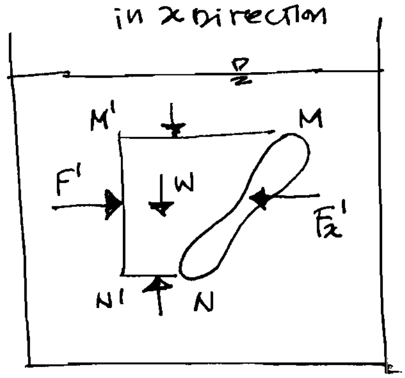
FORCE ON A CURVED SURFACE

Consider pressure forces on a curved surface MN



HORIZONTAL COMPONENT

EQUILIBRIUM in x direction



F_x = NET HORIZONTAL FORCE FROM WATER ON SURFACE MN

F_x' = REACTION FROM SURFACE MN ON WATER

By Newton's 3rd Law

$F_x' = F_x$

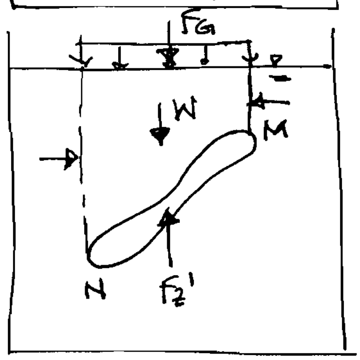
Equilibrium in x-direction:

$F_x' - F' = 0 \Rightarrow F_x' = F$

Also, $\underline{F_x = F_x' = F}$

NOTE: $M'N'$ = vertical projection of curved surface ~~MN~~ MN. Thus, the horizontal component of the force on a curved surface is equal to the force on the vertical projection of the curved surface. $F_x = F'$

VERTICAL COMPONENT



F_G = superimposed gas force at free surface

F_z = net vertical component on curved surface MN (water on surface)

F_z' = reaction from surface onto water

By Newton's 3rd Law, $F_z' = F_z$

Equilibrium in z-direction: $F_z' - W - F_G = 0$, W = weight of water above MN

$\Rightarrow F_z' = W + F_G$, also, $\underline{F_z = F_z' = W + F_G}$

Thus, the vertical component of the force on a curved surface is equal to the weight of liquid ABOVE the curved surface. ~~Whether there is water above or only~~

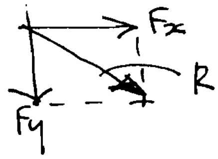
LOCATION OF FORCE COMPONENTS

- Horizontal force: line of action is the same as that of the force on the vertical projection of the curved surface.

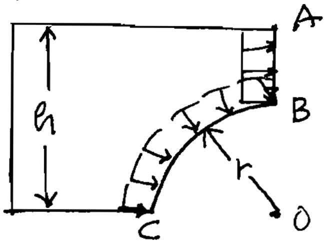
- Vertical force: F_G (gas force) acts through centroid of horizontal projection of curved surface.

  W acts through the center of gravity of the liquid weight

NOTE: if F_x and F_z are in the same plane, a resultant force can be calculated



3.8.2. The cross-section of a tank is as shown in Fig. X.3.8.2. BC is a cylindrical surface with $r = 6$ ft and $h = 10$ ft. If the tank contains gas at a pressure of 8 psi, determine the magnitude and location of the horizontal- and vertical-force components acting on a unit width of tank wall ABC.

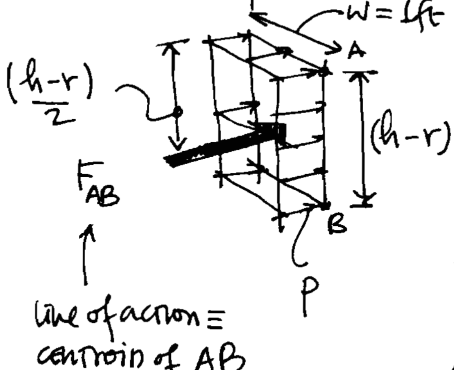


In a gas, the pressure is constant everywhere

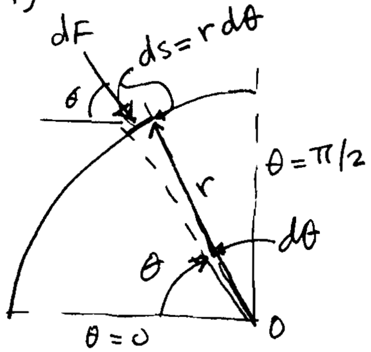
$$p = 8\text{ psi} = 8 \times 144 \frac{\text{lb}}{\text{ft}^2} = 1152 \frac{\text{lb}}{\text{ft}^2}$$

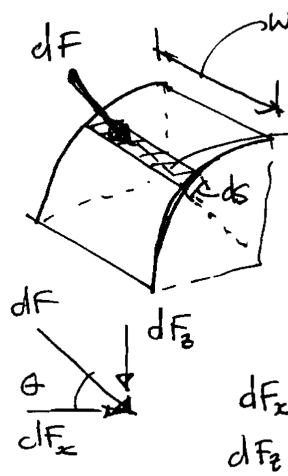
The pressure distribution is shown in the figure.

Thus, force on AB is $F_{AB} = \left(\begin{array}{c}\text{volume of pressure}\\ \text{prism}\end{array}\right) = p \cdot (h-r) \cdot w$



Force on BC





$dA = ds \cdot w$

$dA = w r d\theta$

$dF = p dA$
$= p w r d\theta$

$dF_x = dF \cos\theta$

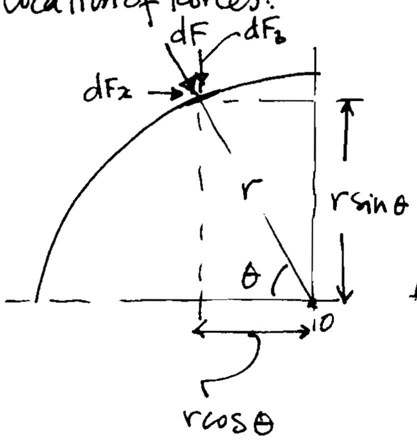
$dF_z = -dF \sin\theta$

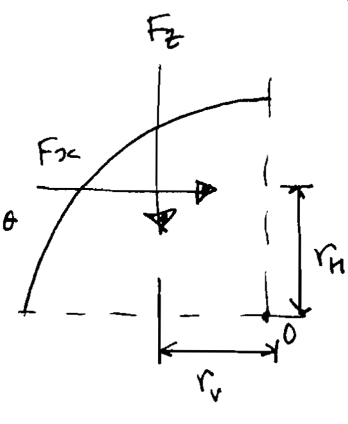
$$dF_x = pwr\cos\theta\, d\theta, \quad dF_z = -pwr\sin\theta\, d\theta$$

$$F_x = \int_0^{\pi/2} dF_x = pwr \int_{\theta=0}^{\theta=\pi/2} \cos\theta\, d\theta = pwr \sin\theta \Big|_0^{\pi/2} = pwr(1-0) = pwr$$

$$F_z = \int_0^{\pi/2} dF_z = -pwr \int_{\theta=0}^{\theta=\pi/2} \sin\theta\, d\theta = -pwr(-\cos\theta)\Big|_0^{\pi/2} = pwr(0-1) = -pwr$$

Location of forces:





$$(\Sigma M_x)_0 = r_H \cdot F_x, \quad (\Sigma M_z)_0 = r_v \cdot F_z$$

$$(\Sigma M_x)_0 = \int_0^{\pi/2} dF_x \cdot r\sin\theta = \int_0^{\pi/2} pwr^2 \sin\theta\cos\theta\, d\theta$$

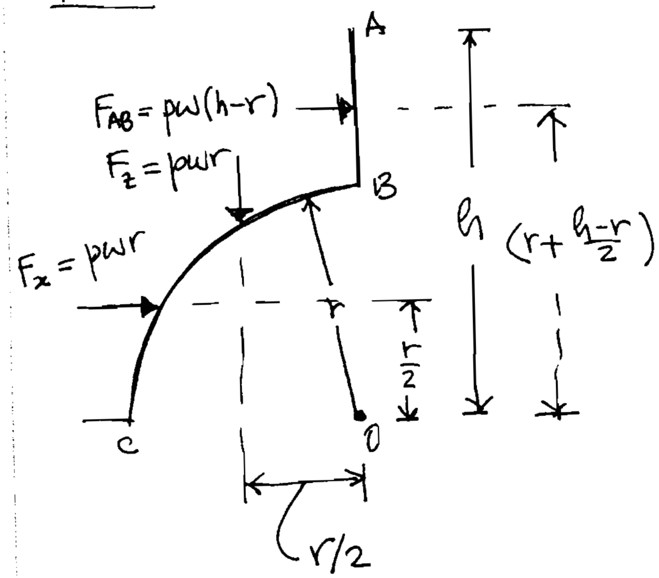
$$= \frac{pwr^2}{2}\int_0^{\pi/2} \sin 2\theta\, d\theta = \frac{pwr^2}{4}\cos 2\theta \Big|_0^{\pi/2}$$

$$= \frac{pwr^2}{4}(-1-1) = -\frac{1}{2}pwr^2$$

$$-\frac{1}{2}pwr^2 = r_v \cdot (-pwr) \Rightarrow \boxed{r_v = \frac{r}{2}}$$

similarly, $\boxed{r_h = r/2}$

FORCES:



TOTAL HORIZONTAL FORCE

$$F_H = F_{AB} + F_z = p w (h - r) + p w r$$

$$= p w h - \cancel{p w r} + \cancel{p w r} = p w h$$

TOTAL VERTICAL FORCE

$$F_V = F_z = p w r$$

line of action of horizontal force

$$r + \frac{h-r}{2} = \frac{2r + h - r}{2} = \frac{h+r}{2}$$

$$r_H' \cdot F_H = F_{AB} \cdot \left(r + \frac{h-r}{2}\right) + F_x \left(\frac{r}{2}\right)$$

$$r_H' \cdot p w h = p w (h - r)\left(\frac{h+r}{2}\right) + p w r \cdot \frac{r}{2}$$

$$r_H' \cdot p w h = \tfrac{1}{2} p w (h^2 - r^2) + \tfrac{1}{2} p w r^2$$

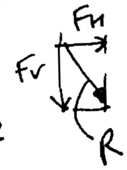
$$r_H' \cdot p w h = \tfrac{1}{2} p w h^2 - \cancel{\tfrac{1}{2} p w r^2} + \cancel{\tfrac{1}{2} p w r^2}$$

$$r_H' = \frac{h}{2}$$

RESULTANT:

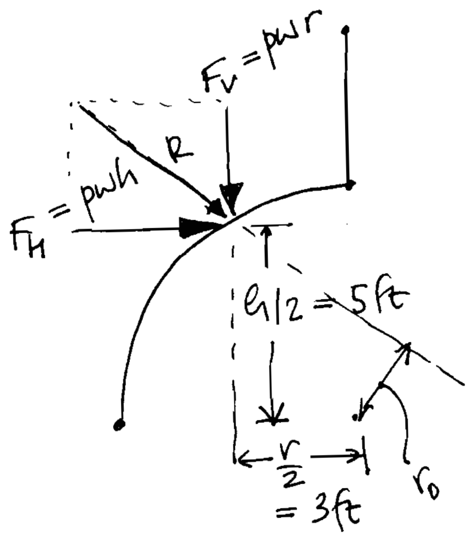
$$R = \sqrt{F_H^2 + F_V^2} = \sqrt{(p w h)^2 + (p w r)^2}$$

$$R = p w \sqrt{h^2 + r^2}$$



NUMERICAL VALUES $p = 8\ psi = 1152\ lb/ft^2$

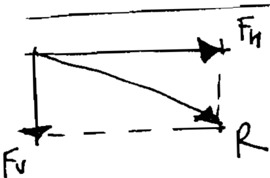
$h = 10\ ft$, $r = 6\ ft$, $w = 1\ ft$



$$F_H = p w h = (1152\ \tfrac{lb}{ft^2})(1ft)(10ft) = 11{,}520\ lb$$

$$F_V = p w r = (1152\ \tfrac{lb}{ft^2})(1ft)(6ft) = 6{,}912\ lb$$

$$R = \sqrt{F_H^2 + F_V^2} = \sqrt{11520^2 + 6912^2} = 13{,}434.5\ lb$$



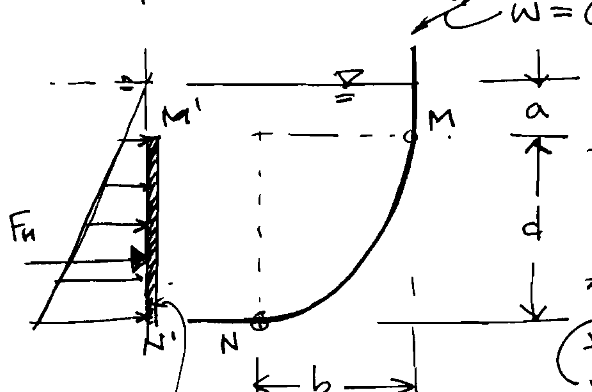
r_0 = arm of R w.r.t. O

$$r_0 \cdot R = \Sigma M_o$$

$$r_0 \cdot 13434.5 = 11520 \times 5 + (-6912) \times 3$$

$$r_0 = \frac{57600 - 20736}{13434.5} = 2.74\ ft$$

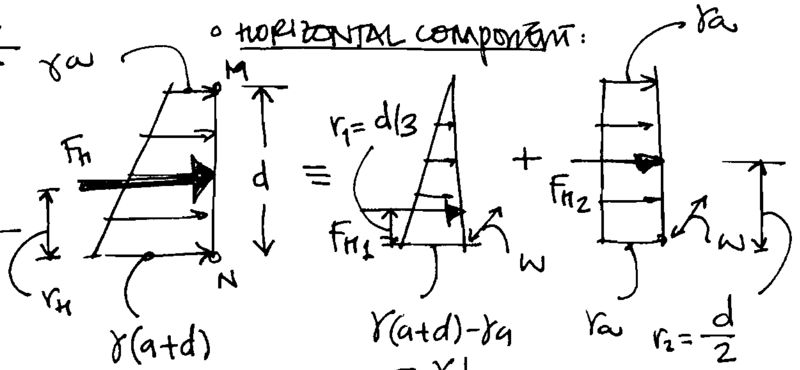
3.8.9. A tank with vertical ends contains water and is 6 m long normal to the plane of Fig. X3.8.9. The sketch shows a proportion of its cross section where MN is one-quarter of an ellipse with semiaxes b and d. If a = 1.0 m, b = 2.5 m, and d = 4 m, find for the surface represented by MN, the magnitude and position of the line of action of (a) the horizontal component of force; (b) the vertical component of force; (c) the resultant force and its direction relative to the horizontal.



VERTICAL PROJECTION OF CURVED SURFACE MN

$a = 1.0\,m,\ b = 2.5\,m,\ d = 4.0\,m$

∘ HORIZONTAL COMPONENT:



$$F_{H_1} = \tfrac{1}{2}(\gamma d)(d)(w) = \tfrac{1}{2}\gamma w d^2$$

$$F_{H_2} = (\gamma a)(d)(w) = \gamma w a d$$

$$\boxed{F_H = F_{H_1} + F_{H_2} = \tfrac{1}{2}\gamma w d^2 + \gamma w a d = \gamma w d\left(\tfrac{d}{2} + a\right)}$$

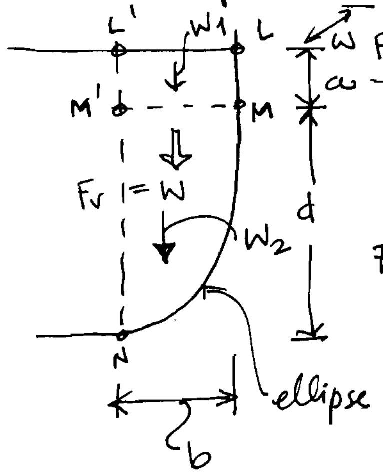
$$\Sigma M_N = 0 \quad F_H \cdot r_H = F_{H_1}\cdot r_1 + F_{H_2}\cdot r_2 \Rightarrow \gamma w d\left(\tfrac{d}{2}+a\right) r_H = \tfrac{1}{2}\gamma w d^2 \cdot \tfrac{d}{3} + \gamma w a d \cdot \tfrac{d}{2}$$

$$r_H \cdot \gamma w d\left(\frac{d+2a}{2}\right) = \frac{1}{6}\gamma w d^3 + \frac{1}{2}\gamma w a d^2$$

$$r_H \cdot \gamma w d\left(\frac{d+2a}{2}\right) = \frac{\gamma w d^3 + 3\gamma w a d^2}{6} \Rightarrow \cancel{\gamma w d}\left(\frac{d+2a}{\cancel{2}}\right) r_H = \cancel{\gamma w} d^2\left(\frac{d+3a}{\cancel{6}\ 3}\right)$$

$$\boxed{r_H = \frac{d}{3}\left(\frac{d+3a}{d+2a}\right)}$$

∘ VERTICAL COMPONENT: WEIGHT OF LIQUID ABOVE MN,



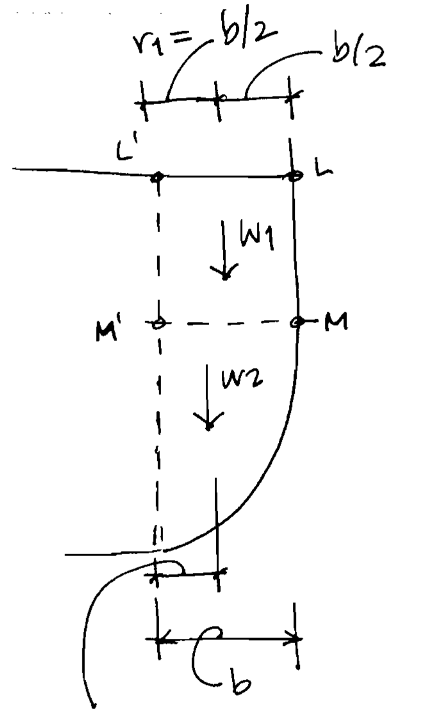
$$F_V = W = \gamma \cdot A \cdot w,\ A = A_{L'LMM'} + A_{M'MN} = ab + \tfrac{\pi}{16}bd = b\left(a + \tfrac{\pi}{16}d\right)$$

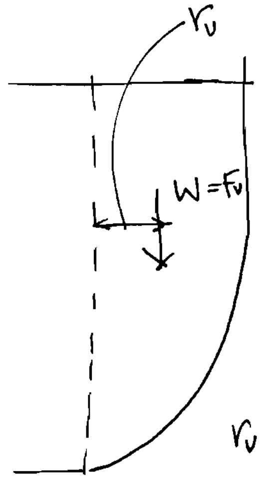
$$A_{L'LM'M} = a\cdot b$$

$$A_{M'MN} = \tfrac{1}{4}(\text{area of ellipse}) = \tfrac{1}{4}\times\tfrac{\pi}{4}bd = \tfrac{\pi}{16}bd$$

$$F_V = \gamma b\left(a + \tfrac{\pi}{16}d\right)w$$

$$W_1 = \gamma A_{LL'MM'}w = \gamma a b w,\ W_2 = \gamma A_{M'MN}w = \tfrac{\pi}{16}\gamma b d w$$





$$F_v = W = \gamma b w\left(a + \frac{\pi}{16} b\right)$$

$$r_v \cdot W = r_1 W_1 + r_2 W_2$$

$$r_v \cdot \gamma b w\left(a+\frac{\pi}{16}b\right) = \frac{b}{2}\gamma a b w + \frac{4b}{3\cancel{\pi}}\,\frac{\cancel{\pi}}{\cancel{16}\,4}\,\gamma b d w$$

$$r_v \cdot \cancel{\gamma b w}\left(\frac{16a+\pi b}{16}\right) = \cancel{\gamma}\, b^{2} \cancel{w}\left(\frac{a}{2} + \frac{d}{12}\right)$$

$$r_v \cdot \frac{16a+\pi b}{\cancel{16}\,4} = \frac{6a+d}{\cancel{12}\,3}$$

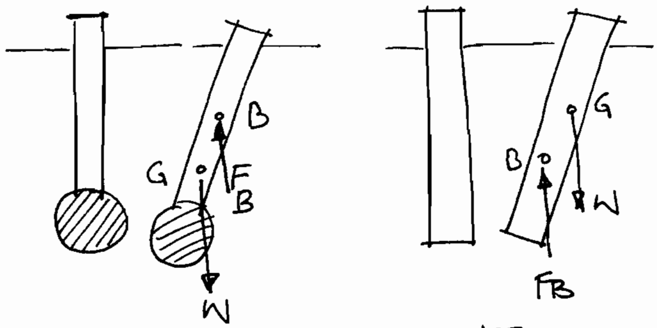
$$\boxed{r_v = \frac{4}{3}\left(\frac{6a+d}{16a+\pi b}\right)}$$

$r_2 = \dfrac{4b}{3\pi}$ (use Table A.7)

$W_2 = \dfrac{\pi}{16}\gamma b d w$

$W_1 = \gamma a b w$

Stability (FLOATING BODY)

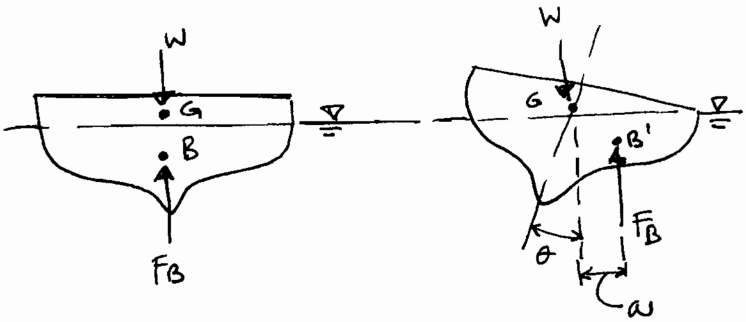


STABLE UNSTABLE

Since the position of B can change as the body tilts (lists), the relative position of G and B is not ~~?~~ a criteria for stability of floating bodies as it was for a submerged body.

The floating body is stable as long as a righting moment develops when the body lists.

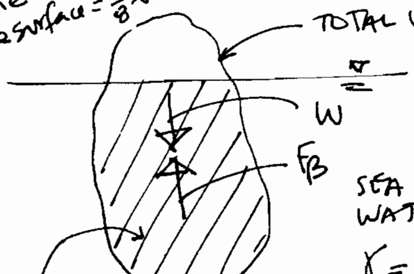
The stabilities of many floating bodies depend upon their shapes.



(SEE Sample Problem 3.9 in pp. 85-86)

Exercises

3.9.3. An iceberg in the ocean floats with 1/8 of its volume above the surface. What is its specific gravity relative to ocean water, which weights 64 lb/ft³? What portion of ~~the~~ its volume would be above ~~water~~ the surface if the ice were floating in pure water?



SEA WATER $\gamma_f = 64\ \mathrm{lb/ft^3}$

submerged volume $= \frac{7}{8}\forall = \forall_{SUB}$

$W = \gamma_s \forall$

$F_B = \gamma_f \forall_{SUB}$

$= \frac{7}{8}\gamma_f \forall$

FOR EQUILIBRIUM

$F_B - W = 0$

$F_B = W$

$\frac{7}{8}\gamma_f \cancel{\forall} = \gamma_s \cancel{\forall}$

$\gamma_s = \frac{7}{8}\gamma_f = \frac{7}{\cancel{8}} \times \overset{8}{\cancel{64}} \frac{\mathrm{lb}}{\mathrm{ft^3}}$

$\boxed{\gamma_s = 56\ \mathrm{lb/ft^3}}$

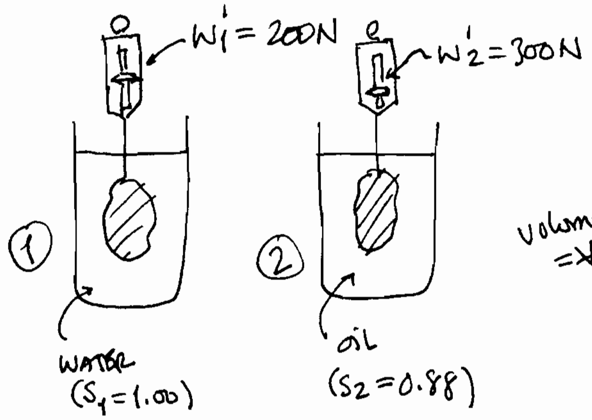
specific gravity relative to ocean water $= \boxed{S_s = \frac{\gamma_s}{\gamma_f} = \frac{7}{8}}$

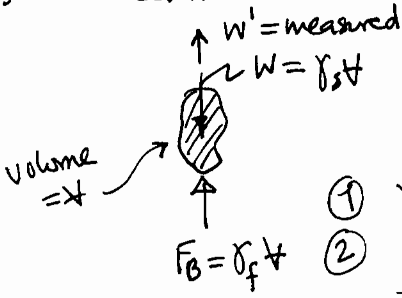
if γ_f is replaced by $\gamma_w = 62.4\ \mathrm{lb/ft^3}$

$W = \gamma_s \forall$, $F_B = \gamma_w \forall_{SUB}$. In equilibrium $F_B = W \Rightarrow \gamma_w \forall_{SUB} = \gamma_s \forall \Rightarrow \frac{\forall_{SUB}}{\forall} = \frac{\gamma_s}{\gamma_w}$

$\frac{\forall_{SUB}}{\forall} = \frac{56\ \mathrm{lb/ft^3}}{62.4\ \mathrm{lb/ft^3}} = 0.897$, $\forall_{above} = \forall - \forall_{SUB} \Rightarrow \boxed{\frac{\forall_{above}}{\forall} = 1 - \frac{\forall_{SUB}}{\forall} = 1 - 0.897 = 0.103 = 10.3\%}$

3.9.4. Determine the volume $\forall$ of an object that weights 200 N in water and 300 N in oil ($s = 0.88$). What is the specific weight of the object?





EQUILIBRIUM $\longrightarrow$ $W' + F_B - W = 0$

$$W - F_B = W'$$

$$\gamma_s \forall - \gamma_f \forall = W'$$

① $\gamma_s \forall - s_1 \gamma_w \forall = W_1'$

② $\gamma_s \forall - s_2 \gamma_w \forall = W_2'$

DIVIDING TERM-BY-TERM

$$\frac{\cancel{\forall}(\gamma_s - s_1\gamma_w)}{\cancel{\forall}(\gamma_s - s_2\gamma_w)} = \frac{W_1'}{W_2'}$$

$$\Rightarrow W_2'(\gamma_s - s_1\gamma_w) = W_1'(\gamma_s - s_2\gamma_w) \Rightarrow \gamma_s W_2' - s_1\gamma_w W_2' = \gamma_s W_1' - s_2 \gamma_w W_1'$$

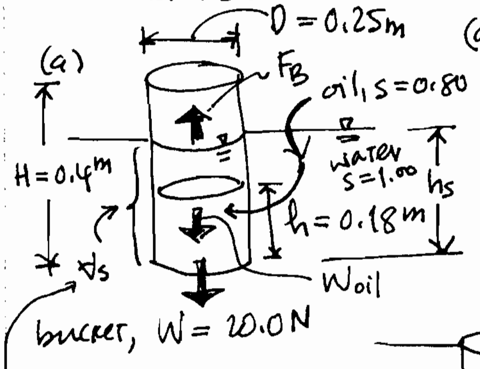
$$\Rightarrow \gamma_s(W_2' - W_1') = \gamma_w(s_1 W_2' - s_2 W_1') \Rightarrow \gamma_s = \gamma_w \cdot \frac{s_1 W_2' - s_2 W_1'}{W_2' - W_1'}$$

combining unit systems

$$\Rightarrow \gamma_s = \left(62.4\frac{lb}{ft^3}\right)\cdot\frac{1\times 300^N - 0.88\times 200^N}{300^N - 200^N} = 62.4\frac{lb}{ft^3}\times\frac{124^N}{100^N} = 77.38\frac{lb}{ft^3}$$

using S.I. only: $\gamma_s = \left(9810\frac{N}{m^3}\right)\times\frac{1\times300^N - 0.88\times200^N}{300^N - 200^N} = 9810\frac{N}{m^3}\times\frac{124^N}{100^N} = 12164.4\ N/m^3$

3.9.7. A cylindrical bucket of 250 mm diameter and 400 mm high weighing 20.0 N contains oil ($s = 0.80$) to a depth of 180 mm. (a) When placed to float in water, what will be the immersion depth to the bottom of the bucket? (b) What is the maximum volume of oil the bucket can hold and still float?



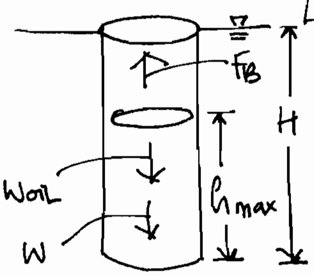
(a) EQUILIBRIUM: $F_B - W - W_{oil} = 0$

$$\gamma_w \frac{\pi D^2}{4}h_s - W - \gamma_{oil}\cdot\frac{\pi D^2}{4}h = 0$$

$$\pi D^2 \gamma_w h_s - 4W - \pi D^2 S_{oil}\cdot\gamma_w\cdot h = 0$$

$$h_s = \frac{4W}{\pi D^2 \gamma_w} + S_{oil}\cdot h = \frac{4\times 20^N}{\pi\times(0.25m)^2(9810N/m^3)} + 0.80\times0.18^m$$

$$\boxed{h_s = 0.042^m + 0.18^m = 0.222m = 222\ mm}$$



(b) about to be submerged: $F_B - W - W_{oil} = 0$ with

$$W_{oil} = \frac{\pi D^2}{4}h_{max}\gamma_{oil} = \frac{1}{4}\pi D^2 h_{max} S_{oil}\gamma_w$$

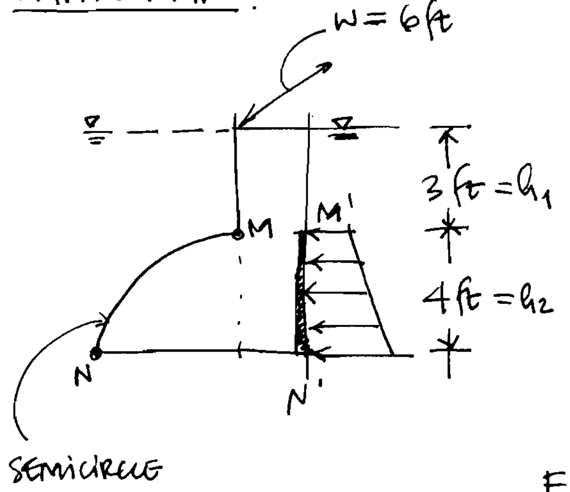
$$F_B = \frac{\pi D^2}{4}H\gamma_w = \frac{1}{4}\pi D^2 H \gamma_w$$

$$\frac{\pi D^2}{\cancel{4}}H\gamma_w - 4W - \frac{1}{\cancel{4}}\pi D^2 h_{max} S_{oil}\gamma_w = 0$$

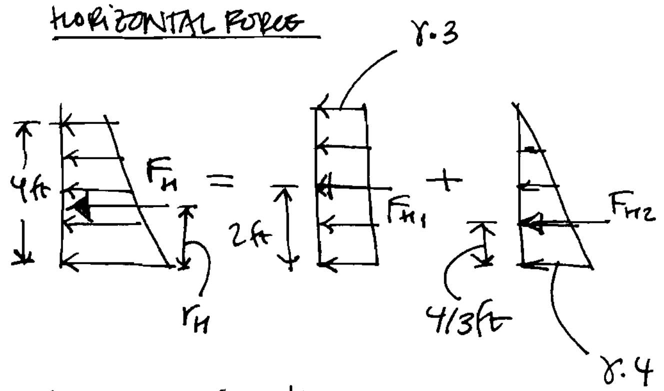
NOTE: since $h_{max} > H$, the cylinder never submerges

$$h_{max} = -\frac{4W}{\pi D^2 S_{oil}\gamma_w} + \frac{H}{S_{oil}} = -\frac{4\times20}{\pi\times0.25^2\times0.88\times9810} + \frac{0.4}{0.88} = -0.047 + 0.456 = 0.408 = 408mm$$

EXTRA EXAMPLE —— FIND F_H, F_V ON MN



HORIZONTAL FORCE



$F_{H_1} = \gamma\cdot 3\cdot 4\cdot 6 = (62.4\,\frac{lb}{ft^3})(3ft)(4ft)(6ft) = 4492.8\,lb$

$F_{H_2} = \frac{1}{2}(\gamma\cdot 4)(4)(6) = \frac{1}{2}(62.4\,\frac{lb}{ft^3})(4ft)(4ft)(6ft) = 2995.2\,lb$

$\boxed{F_H = F_{H_1} + F_{H_2} = 7488\,lb}$

$r_H\cdot F_H = 2\times F_{H_1} + \frac{4}{3}\times F_{H_2}$

$r_H\cdot 7488 = 2\times 4492.8 + \frac{4}{3}\times 2995.2$

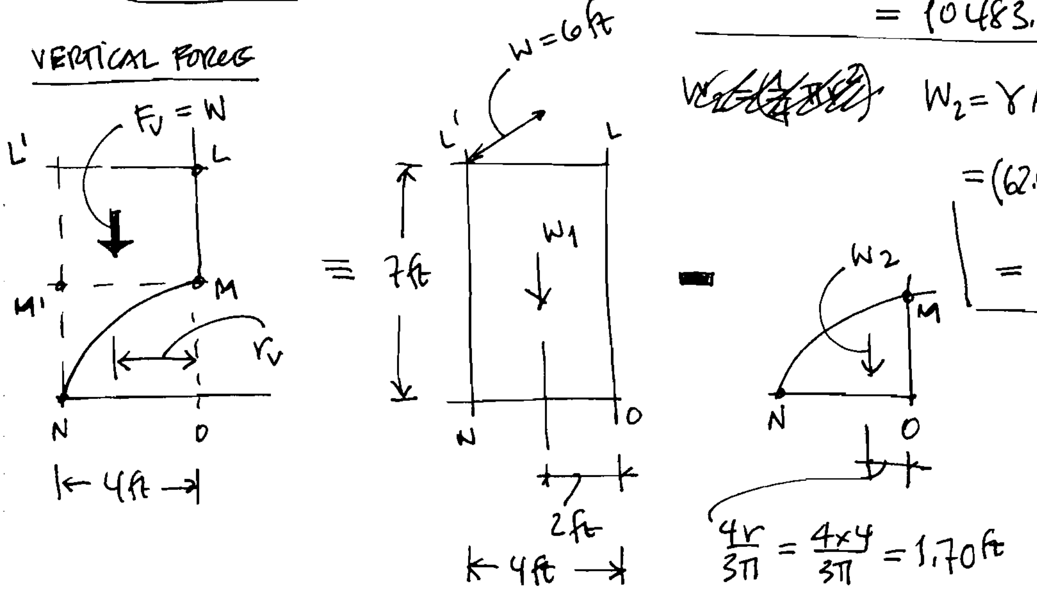
$r_H\cdot 7488 = 8985.6 + 3993.6$

$\boxed{r_H = \frac{12979.2}{7488} = 1.733\,ft}$

$W_1 = \gamma A_{L'LNO}\cdot w = (62.4\,\frac{lb}{ft^3})(4ft)(7ft)(6ft)$

$= 10483.2\,lb$

VERTICAL FORCE



$\frac{4r}{3\pi} = \frac{4\times 4}{3\pi} = 1.70\,ft$

$W_2 = \gamma A_{NMO}\,w = \gamma\cdot\frac{1}{4}\pi r^2 w$

$= (62.4\,\frac{lb}{ft^3})\,\frac{1}{4}\pi\times(4ft)^2\times(6ft)$

$= 4704.85\,lb$

$\boxed{F_V = W_1 + W_2 = 10483.2\,lb - 4704.85\,lb = \cancel{15188.05\,lb}}$ — 5778.35

$r_V\cdot F_V = 2\times W_1 - 1.70\times W_2 \Rightarrow r_V\cdot 15188.05 = 2\times 10483.2 - 1.70\times 4704.85$

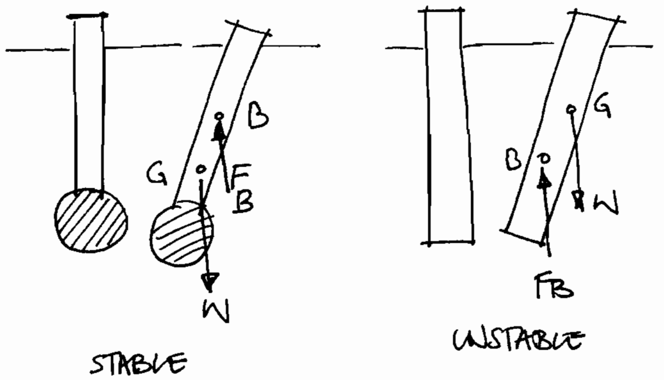
$\cancel{15188.05}$ 5778.35 $r_V = 20966.4 - 7998.25$

$5778.35\,r_V = 12968.15$

~~15188.05 r_V = 28964.65~~ ⇒

$\boxed{r_V = 2.24\,ft}$

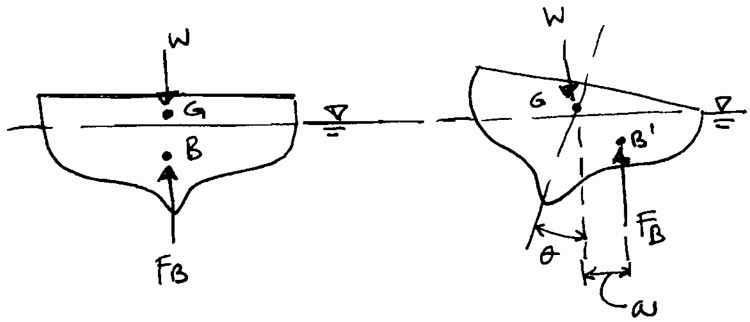
Stability (FLOATING BODY)



Since the position of B can change as the body tilts (lists), the relative position of G and B is not ~~ ~~ a criteria for stability of floating bodies as it was for a submerged body.

The floating body is stable as long as a righting moment develops when the body lists.

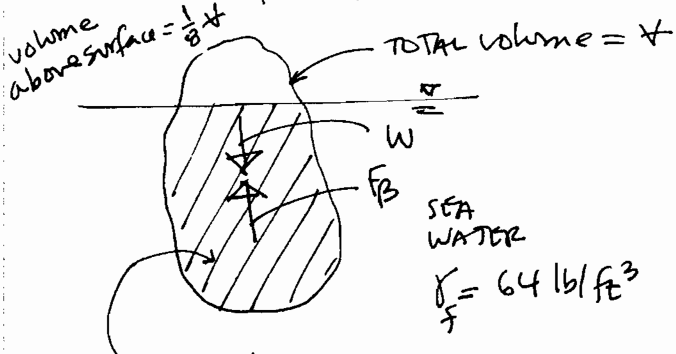
The stabilities of many floating bodies depend upon their shapes.



(SEE Sample Problem 3.9 in pp. 85-86)

Exercises

3.9.3. An iceberg in the ocean floats with 1/8 of its volume above the surface. What is its specific gravity relative to ocean water, which weights 64 lb/ft^3? What portion of ~~the~~ its volume would be above ~~water~~ the surface if the ice were floating in pure water?



$\gamma_f = 64\ \text{lb/ft}^3$

submerged volume $= \frac{7}{8}V = V_{SUB}$

$W = \gamma_s V$

$F_B = \gamma_f V_{SUB}$

$= \frac{7}{8}\gamma_f V$

FOR Equilibrium

$F_B - W = 0$

$F_B = W$

$\frac{7}{8}\gamma_f \cancel{V} = \gamma_s \cancel{V}$

$\gamma_s = \frac{7}{8}\gamma_f = \frac{7}{\cancel{8}} \times \overset{8}{\cancel{64}}\ \frac{\text{lb}}{\text{ft}^3}$

$\boxed{\gamma_s = 56\ \text{lb/ft}^3}$

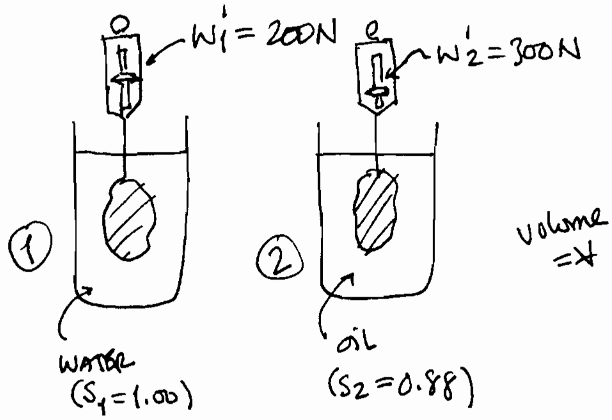
specific gravity relative to ocean water $= \boxed{S_s = \frac{\gamma_s}{\gamma_f} = \frac{7}{8}}$

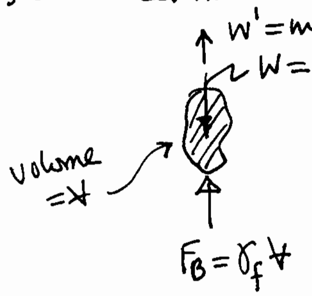
if γ_f is replaced by $\gamma_w = 62.4\ \text{lb/ft}^3$

$W = \gamma_s V$, $F_B = \gamma_w V_{SUB}$. In equilibrium $F_B = W \Rightarrow \gamma_w V_{SUB} = \gamma_s V \Rightarrow \frac{V_{SUB}}{V} = \frac{\gamma_s}{\gamma_w}$

$\frac{V_{SUB}}{V} = \frac{56\ \text{lb/ft}^3}{62.4\ \text{lb/ft}^3} = 0.897$, $V_{above} = V - V_{SUB} \Rightarrow \boxed{\frac{V_{above}}{V} = 1 - \frac{V_{SUB}}{V} = 1 - 0.897 = 0.103 = 10.3\%}$

3.9.4. Determine the volume $\forall$ of an object that weights 200 N in water and 300 N in oil (s = 0.88). What is the specific weight of the object?





Equilibrium $\longrightarrow$ $W' + F_B - W = 0$

$W - F_B = W'$

$\gamma_s \forall - \gamma_f \forall = W'$

① $\gamma_s \forall - s_1 \gamma_w \forall = W_1'$

② $\gamma_s \forall - s_2 \gamma_w \forall = W_2'$

DIVIDING TERM-BY-TERM

$$\frac{\cancel{\forall}(\gamma_s - s_1 \gamma_w)}{\cancel{\forall}(\gamma_s - s_2 \gamma_w)} = \frac{W_1'}{W_2'}$$

$$\Rightarrow W_2'(\gamma_s - s_1\gamma_w) = W_1'(\gamma_s - s_2\gamma_w) \Rightarrow \gamma_s W_2' - s_1 \gamma_w W_2' = \gamma_s W_1' - s_2 \gamma_w W_1'$$

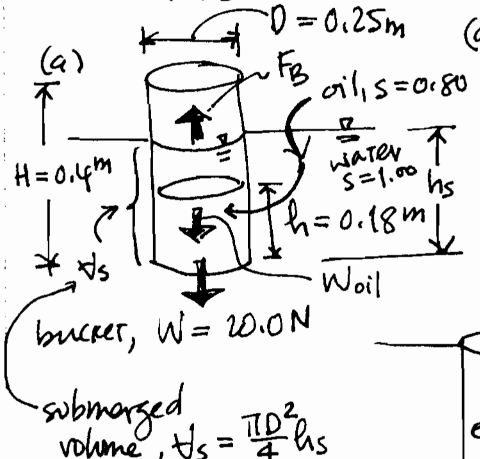
$$\Rightarrow \gamma_s(W_2' - W_1') = \gamma_w(s_1 W_2' - s_2 W_1') \Rightarrow \gamma_s = \gamma_w \cdot \frac{s_1 W_2' - s_2 W_1'}{W_2' - W_1'}$$

combining unit systems

$$\Rightarrow \gamma_s = \left(62.4 \frac{lb}{ft^3}\right) \cdot \frac{1 \times 300^N - 0.88 \times 200^N}{300^N - 200^N} = 62.4 \frac{lb}{ft^3} \times \frac{124^N}{100^N} = 77.38 \frac{lb}{ft^3}$$

using S.I. only: $\gamma_s = \left(9810 \frac{N}{m^3}\right) \times \frac{1 \times 300^N - 0.88 \times 200^N}{300N - 200N} = 9810 \frac{N}{m^3} \times \frac{124^N}{100^N} = 12164.4\ N/m^3$

3.9.7. A cylindrical bucket of 250 mm diameter and 400 mm high weighing 20.0 N contains oil (s = 0.80) to a depth of 180 mm. (a) When placed to float in water, what will be the immersion depth to the bottom of the bucket? (b) What is the maximum volume of oil the bucket can hold and still float?



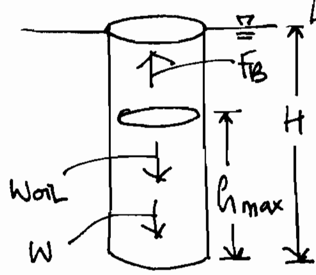
(a) EQUILIBRIUM: $F_B - W - W_{OIL} = 0$

$$\gamma_w \frac{\pi D^2}{4} h_s - W - \gamma_{OIL} \cdot \frac{\pi D^2}{4} h = 0$$

$$\pi D^2 \gamma_w h_s - 4W - \pi D^2 S_{OIL} \cdot \gamma_w \cdot h = 0$$

$$h_s = \frac{4W}{\pi D^2 \gamma_w} + S_{oil} \cdot h = \frac{4 \times 20^N}{\pi \times (0.25_m)^2 (9810 N/m^3)} + 0.80 \times 0.18^m$$

$$\boxed{h_s = 0.042^m + 0.18^m = 0.222 m = 222 mm}$$



(b) about to be submerged: $F_B - W - W_{OIL} = 0$ with

$$W_{OIL} = \frac{\pi D^2}{4} h_{max} \gamma_{OIL} = \frac{1}{4} \pi D^2 h_{max} S_{oil} \gamma_w$$

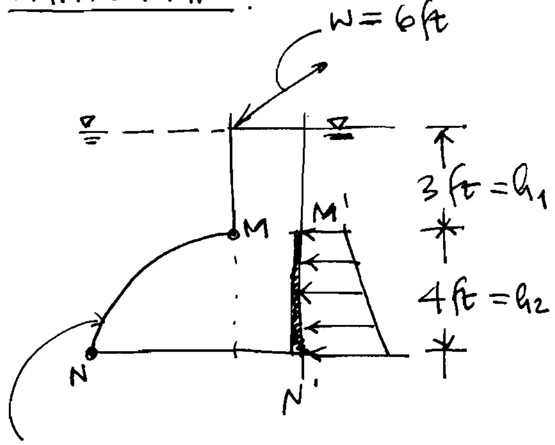
$$F_B = \frac{\pi D^2}{4} H \gamma_w = \frac{1}{4} \pi D^2 H \gamma_w$$

$$\frac{\pi D^2}{\cancel{4}} H \gamma_w \ 4W - \frac{1}{\cancel{4}} \pi D^2 h_{max} S_{oil} \gamma_w = 0$$

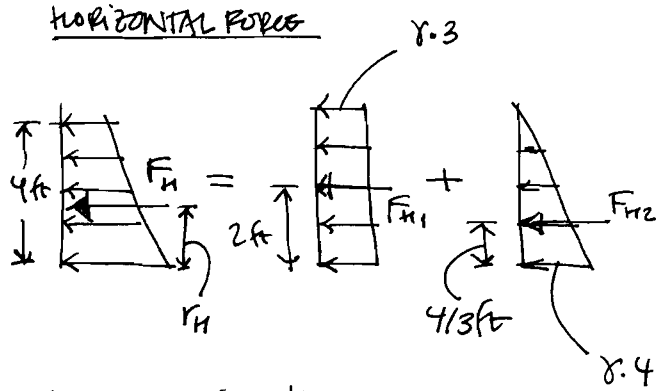
NOTE: since $h_{max} > H$, the cylinder never submerges

$$h_{max} = -\frac{4W}{\pi D^2 S_{oil} \gamma_w} + \frac{H}{S_{OIL}} = -\frac{4 \times 20}{\pi \times 0.25^2 \times 0.88 \times 9810} + \frac{0.4}{0.88} = -0.047 + 0.455 = 0.408 = 408 mm$$

EXTRA EXAMPLE — FIND F_H, F_V ON MN



HORIZONTAL FORCE



$$F_{H_1} = \gamma\cdot 3\cdot 4\cdot 6 = \left(62.4\,\frac{lb}{ft^3}\right)(3ft)(4ft)(6ft) = 4492.8\,lb$$

$$F_{H_2} = \frac{1}{2}(\gamma\cdot 4)(4)(6) = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(4ft)(4ft)(6ft) = 2995.2\,lb$$

$$\boxed{F_H = F_{H_1} + F_{H_2} = 7488\,lb}$$

$$r_H\cdot F_H = 2\times F_{H_1} + \frac{4}{3}\times F_{H_2}$$

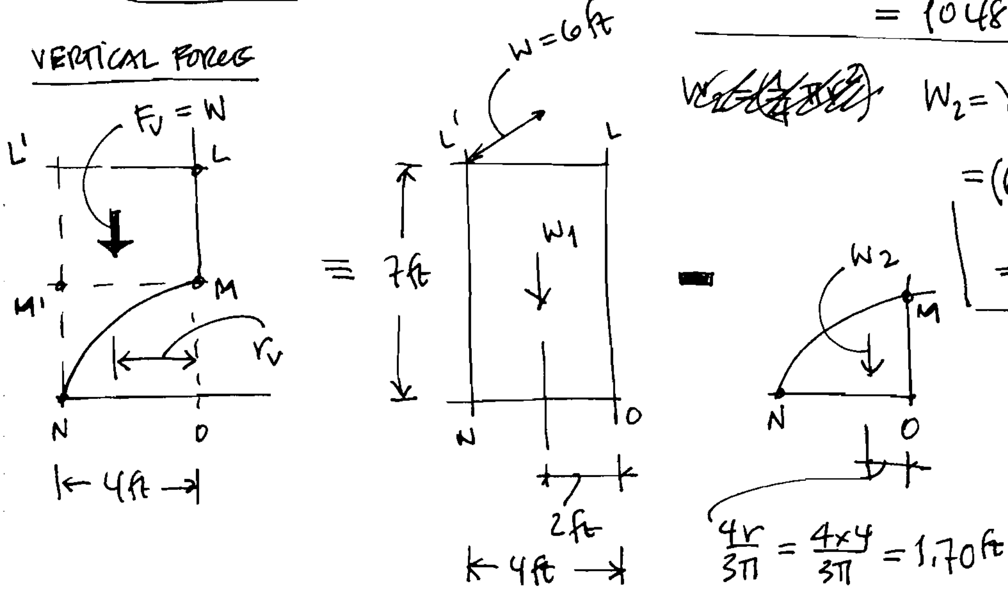
$$r_H\cdot 7488 = 2\times 4492.8 + \frac{4}{3}\times 2995.2$$

$$r_H\cdot 7488 = 8985.6 + 3993.6$$

$$\boxed{r_H = \frac{12979.2}{7488} = 1.733\,ft}$$

$$W_1 = \gamma A_{L'LNO}\cdot w = \left(62.4\,\frac{lb}{ft^3}\right)(4ft)(7ft)(6ft)$$
$$= 10483.2\,lb$$

VERTICAL FORCE



$\cancel{W_2 = \ldots}$ $W_2 = \gamma A_{NMO}\, w = \gamma\cdot\frac{1}{4}\pi r^2 w$

$$= \left(62.4\,\frac{lb}{ft^3}\right)\frac{1}{4}\pi\times(4ft)^2\times(6ft)$$

$$= 4704.85\,lb$$

$$\frac{4r}{3\pi} = \frac{4\times 4}{3\pi} = 1.70\,ft$$

$$\boxed{F_V = W_1 + W_2 = 10483.2\,lb \cancel{+} 4704.85\,lb = \cancel{15188.05\,lb}}\ 5778.35$$

$$r_V\cdot F_V = 2\times W_1 \cancel{+} 1.70\times W_2 \Rightarrow r_V\cdot 15188.05 = 2\times 10483.2 \cancel{+} 1.70\times 4704.85$$

$$\cancel{15188.05}\ 5778.35\ r_V = 20966.4 \cancel{+} 7998.25$$

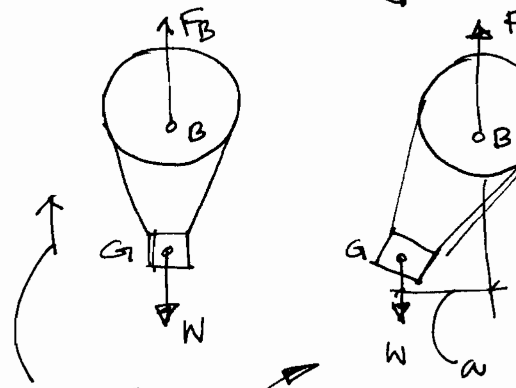
$$5778.35\ r_V = 12968.15$$

$$\cancel{15188.05\ r_V = 28964.65} \Rightarrow \cancel{\ldots}$$

$$\boxed{r_V = 2.24\,ft}$$

Stability (submerged body)

giving the system a tilt



Righting moment: $W \times a$ if $W < F_B$

$F_B \times a$ if $W > F_B$

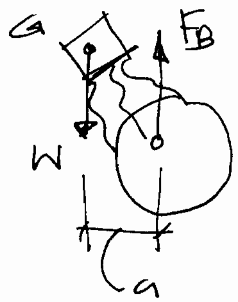
TENDS TO RESTORE BODY TO ORIGINAL (STABLE) POSITION

G = center of gravity

B = center of buoyancy

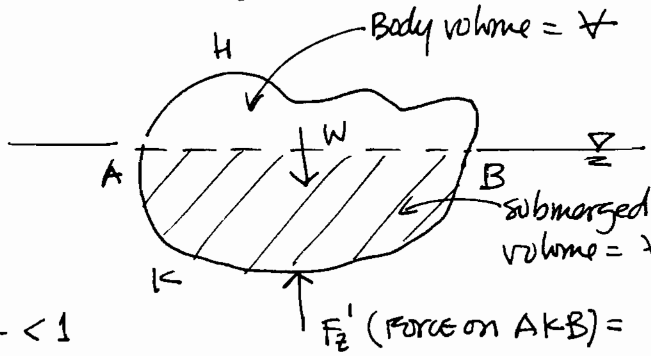
NET LIFT $= F_B - W$

If the center of buoyancy is above the center of gravity the body is stable



unstable

Floating Body

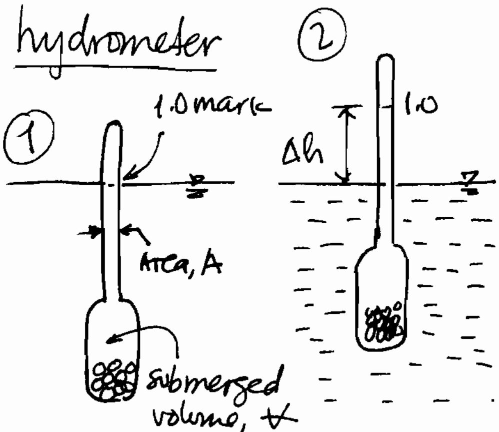


submerged volume = $\forall_s$

F_z' (Force on AKB) = F_B (weight of ABK)

$$W = F_B \Rightarrow \gamma_s \forall = \gamma_f \forall' \Rightarrow \frac{\forall'}{\forall} = \frac{\gamma_s}{\gamma_f} < 1$$

hydrometer



$S = 1.0$ (i.e. water)

$S > 1.0$ (denser liquid)

In water, submerged volume $= \forall$

In denser liquid, sub. volume $= \forall - A\Delta h$

Hydrometer's weight, $W = \gamma_w \forall$ (1)

$$W = s\gamma_w(\forall - A\Delta h) \quad (2)$$

$$\cancel{\gamma_w}\forall = s\cancel{\gamma_w}\forall - s\cancel{\gamma_w}A\Delta h$$

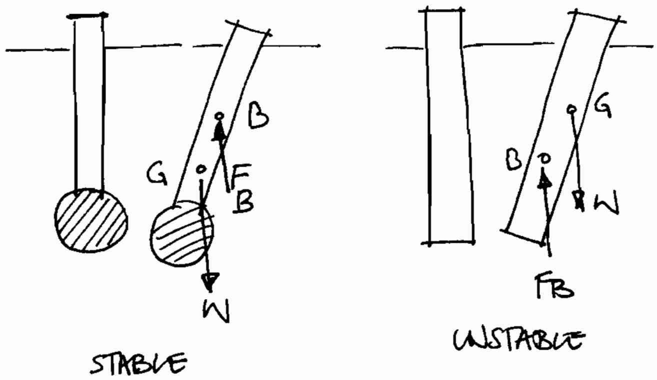
$$sA\Delta h = (s-1)\forall \Rightarrow$$

$$\boxed{\Delta h = \frac{\forall}{A}\left(\frac{s-1}{s}\right)}$$

use this relationship to mark the hydrometer

instrument used to measure the specific gravity of a liquid

Stability — FLOATING BODY



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The stabilities of many floating bodies depend upon their shapes.

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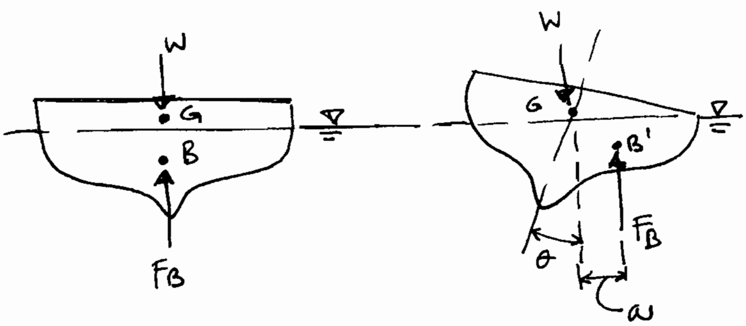

Exercises

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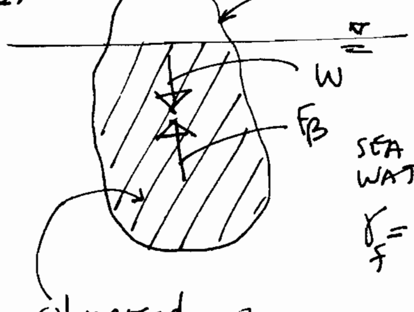
volume above surface $= \frac{1}{8}\forall$

TOTAL volume $= \forall$



SEA WATER

$\gamma_f = 64\ \text{lb/ft}^3$

submerged volume $= \frac{7}{8}\forall = \forall_{SUB}$

$W = \gamma_s \forall$

$F_B = \gamma_f \forall_{SUB}$

$= \frac{7}{8}\gamma_f \forall$

FOR EQUILIBRIUM

$F_B - W = 0$

$F_B = W$

$\frac{7}{8}\gamma_f \cancel{\forall} = \gamma_s \cancel{\forall}$

$\gamma_s = \frac{7}{8}\gamma_f = \frac{7}{8} \times 64\ \frac{lb}{ft^3}$

$\boxed{\gamma_s = 56\ \text{lb/ft}^3}$

specific gravity relative to ocean water $= \boxed{s_s = \frac{\gamma_s}{\gamma_f} = \frac{7}{8}}$

if γ_f is replaced by $\gamma_w = 62.4\ \text{lb/ft}^3$

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$\frac{\forall_{SUB}}{\forall} = \frac{56\ \text{lb/ft}^3}{62.4\ \text{lb/ft}^3} = 0.897$, $\forall_{above} = \forall - \forall_{SUB} \Rightarrow \boxed{\frac{\forall_{above}}{\forall} = 1 - \frac{\forall_{SUB}}{\forall} = 1 - 0.897 = 0.103 = 10.3\%}$

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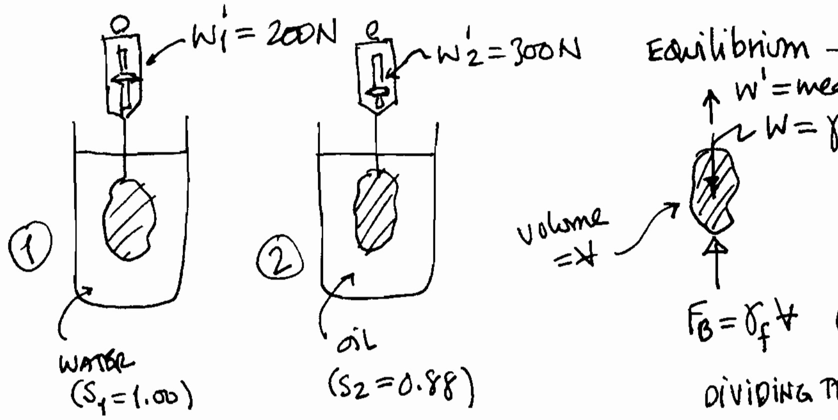

Equilibrium $\longrightarrow$ $W' + F_B - W = 0$

$$W - F_B = W'$$

$$\gamma_s \forall - \gamma_f \forall = W'$$

① $\gamma_s \forall - s_1 \gamma_w \forall = W_1'$

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DIVIDING TERM-BY-TERM

$$\frac{\cancel{\forall}(\gamma_s - s_1\gamma_w)}{\cancel{\forall}(\gamma_s - s_2\gamma_w)} = \frac{W_1'}{W_2'}$$

$$\Rightarrow W_2'(\gamma_s - s_1\gamma_w) = W_1'(\gamma_s - s_2\gamma_w) \Rightarrow \gamma_s W_2' - s_1\gamma_w W_2' = \gamma_s W_1' - s_2 \gamma_w W_1'$$

$$\Rightarrow \gamma_s(W_2' - W_1') = \gamma_w(s_1 W_2' - s_2 W_1') \Rightarrow \gamma_s = \gamma_w \cdot \frac{s_1 W_2' - s_2 W_1'}{W_2' - W_1'}$$

combining unit systems

$$\Rightarrow \gamma_s = \left(62.4 \frac{lb}{ft^3}\right) \cdot \frac{1 \times 300^N - 0.88 \times 200^N}{300^N - 200^N} = 62.4 \frac{lb}{ft^3} \times \frac{124^N}{100^N} = 77.38 \frac{lb}{ft^3}$$

using S.I. only: $\gamma_s = \left(9810 \frac{N}{m^3}\right) \times \frac{1 \times 300^N - 0.88 \times 200N}{300N - 200N} = 9810 \frac{N}{m^3} \times \frac{124^N}{100N} = 12164.4 \, N/m^3$

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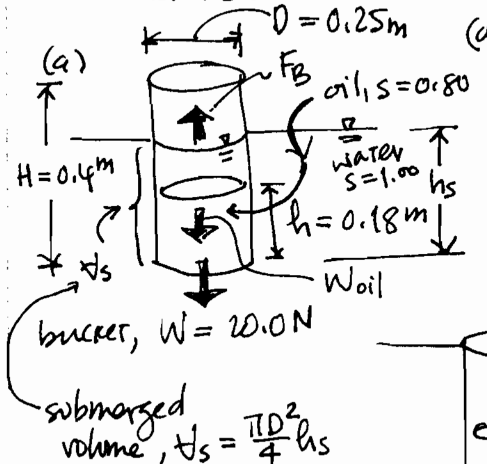

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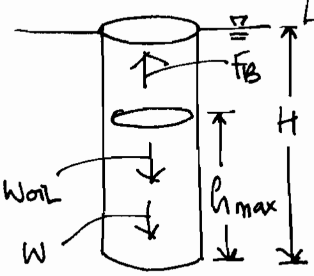

(b) about to be submerged: $F_B - W - W_{oil} = 0$ with

$$W_{oil} = \frac{\pi D^2}{4} h_{max} \gamma_{oil} = \frac{1}{4}\pi D^2 h_{max} S_{oil} \gamma_w$$

$$F_B = \frac{\pi D^2}{4} H \gamma_w = \frac{1}{4}\pi D^2 H \gamma_w$$

$$\frac{\pi D^2}{4} H \gamma_w - W - \frac{1}{4}\pi D^2 h_{max} S_{oil} \gamma_w = 0$$

NOTE: since $h_{max} > H$, the cylinder never submerges

$$h_{max} = -\frac{4W}{\pi D^2 S_{oil} \gamma_w} + \frac{H}{S_{oil}} = -\frac{4 \times 20}{\pi \times 0.25^2 \times 0.88 \times 9810} + \frac{0.4}{0.88} = -0.047 + 0.456 = 0.408 = 408mm$$