

Problems for Chapter 4

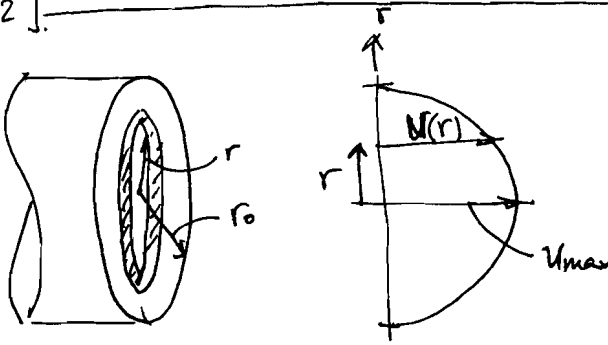
Basics of Fluid Flow

[1]. $s=1.75$ is not needed. $D=3\text{ in} = \frac{3}{12}\text{ ft} = 0.25\text{ ft}$, $V=1.2\text{ ft/s}$

(a) $Q = V \cdot A = V \cdot \frac{\pi D^2}{4} = (1.2 \frac{\text{ft}}{\text{s}}) \left(\frac{\pi \times (0.25\text{ ft})^2}{4} \right) = 0.0589\text{ ft}^3/\text{s}$

(b) if $D=6\text{ in} = 0.5\text{ ft}$, $V = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = \frac{4Q}{\pi D^2} = \frac{4 \times 0.0589\text{ ft}^3/\text{s}}{\pi \times (0.5\text{ ft})^2} = 0.3\text{ ft/s}$

[2].



$$u(r) = u_{\max} \left[1 - \left(\frac{r}{r_0} \right)^2 \right]$$

↑ velocity distribution, function of r

⇒ at constant r , $u(r)$ is the same in the area increment shown (a ring) with area $dA = 2\pi r dr$

Thus, discharge through dA is:

$$dQ = u(r) dA = u_{\max} \left[1 - \left(\frac{r}{r_0} \right)^2 \right] \cdot 2\pi r dr$$

and the total discharge is

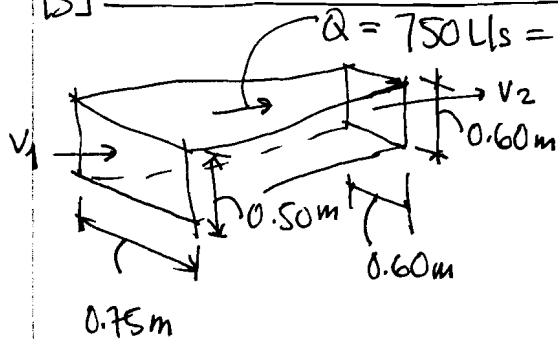
$$Q = \int_A dQ = \int_0^{r_0} 2\pi u_{\max} \left[1 - \left(\frac{r}{r_0} \right)^2 \right] r dr$$

using HP calculator or TI calculator ⇒ e.g., in EQW for HP calculator

$$\int_0^{r_0} 2 \cdot \pi \cdot u_{\max} \cdot \left[1 - \left[\frac{r}{r_0} \right]^2 \right] \cdot r dr \quad \text{EVAL} \quad \Rightarrow \quad Q = \frac{\pi u_{\max} (3r_0^2)}{5} = \frac{3}{5} \pi u_{\max} r_0^2$$

ⓑ with $A = \pi r_0^2$, $V = \frac{Q}{A} = \frac{\frac{3}{5} \pi u_{\max} r_0^2}{\pi r_0^2} = \frac{3}{5} u_{\max}$

[3]



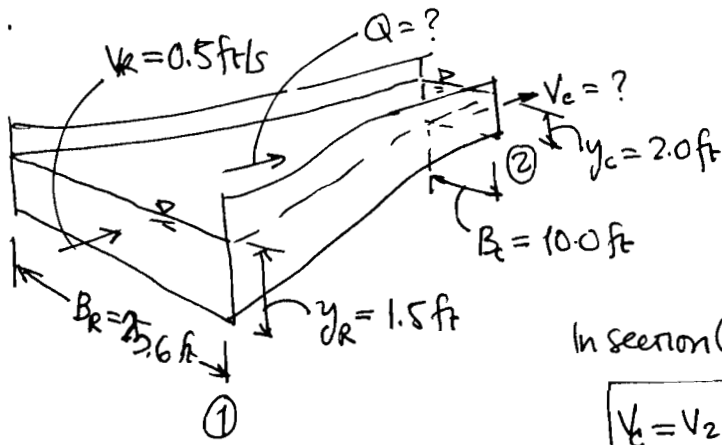
$$A_1 = 0.75 \times 0.50\text{ m}^2 = 0.375\text{ m}^2$$

$$A_2 = (0.60\text{ m})^2 = 0.36\text{ m}^2$$

(a) $V_1 = \frac{Q}{A_1} = \frac{0.75\text{ m}^3/\text{s}}{0.375\text{ m}^2} = 2.00\text{ m/s}$

(b) $V_2 = \frac{Q}{A_2} = \frac{0.75\text{ m}^3/\text{s}}{0.36\text{ m}^2} = 2.08\text{ m/s}$

[4].



In section ① $v_1 = v_2 = 0.5 \text{ ft/s}$

$$A_1 = B_1 y_1 = 25.6 \times 1.5 \text{ ft}^2 = 38.4 \text{ ft}^2$$

$$Q = v_1 A_1 = (0.5 \frac{\text{ft}}{\text{s}})(38.4 \text{ ft}^2)$$

$$\boxed{Q = 19.2 \text{ cfs}}$$

In section ②, $v_2 = v_c = ?$, $A_2 = 10 \times 2 \text{ ft}^2 = 20 \text{ ft}^2$

$$\boxed{v_2 = v_c = \frac{Q}{A_2} = \frac{19.2 \text{ cfs}}{20 \text{ ft}^2} = 0.96 \text{ fps}}$$

[5]

$$Q_1 = 80 \text{ cfs}$$

$$Q_2 = 65 \text{ cfs}$$

$$(a) \frac{dV}{dt} = Q_1 - Q_2 = 80 - 65 = 15 \text{ cfs}$$

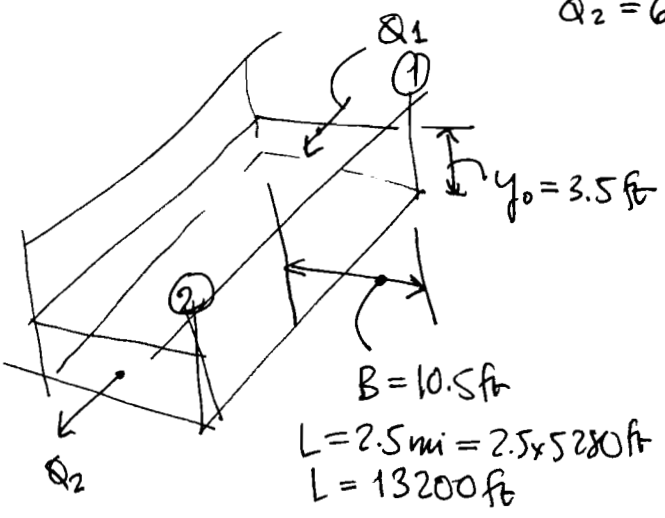
(b) For $B = 10.5 \text{ ft}$, $y_0 = 3.5 \text{ ft}$

$$A_1 = A_2 = B y_0 = 10.5 \times 3.5$$

$$= 36.75 \text{ ft}^2$$

$$v_1 = \frac{Q_1}{A_1} = \frac{80 \text{ cfs}}{36.75 \text{ ft}^2} = 2.17 \text{ fps}$$

$$v_2 = \frac{Q_2}{A_2} = \frac{65 \text{ cfs}}{36.75 \text{ ft}^2} = 1.77 \text{ fps}$$



$$(c) V = B y L \Rightarrow \frac{dV}{dt} = B L \frac{dy}{dt} \Rightarrow 15 = 10.5 \times 13200 \times \frac{dy}{dt}$$

For $\Delta t = 15 \text{ min} = 15 \times 60 = 900 \text{ s}$

$$\Rightarrow \frac{dy}{dt} = 1.08 \times 10^{-4} \frac{\text{ft}}{\text{s}} \approx \frac{\Delta y}{\Delta t} \Rightarrow \Delta y = (1.08 \times 10^{-4})(900) = 0.097 \text{ ft}$$

$$\underline{y = y_0 + \Delta y = 3.5 + 0.097 = 3.597 \text{ ft}}$$