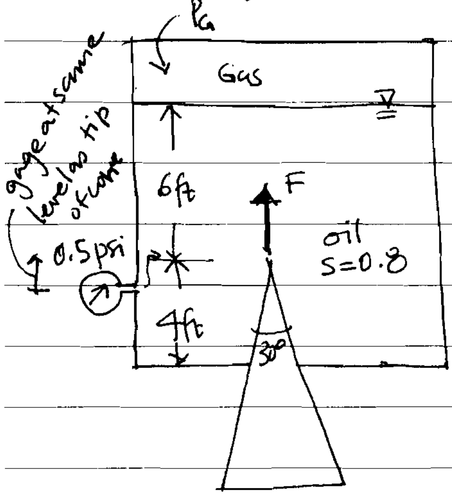
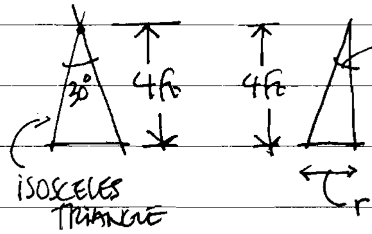
3.8.5. Determine the force F required to hold the cone in the position shown in Fig. X3.8.5. Assume the cone is weightless.




solution: Force F must be equal to the vertical force applied by the liquid and the gas on the free surface

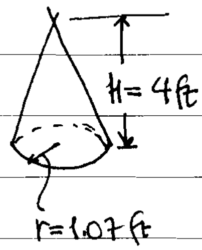



$\tan 15° = \dfrac{r}{4ft}$

$r = 4ft \tan 15°$

$r = 1.07 ft$

$\boxed{\text{volume of cone}}$

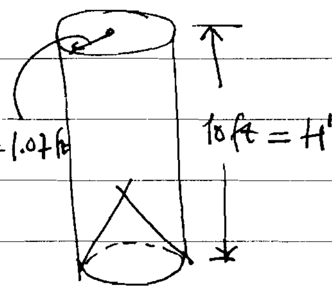



$V_c = \dfrac{\pi r^2 H}{3}$

$V_c = \dfrac{\pi \times 1.07^2 \times 4}{3}$

$\underline{V_c = 4.80\ ft^3}$

$\boxed{\text{volume of oil above cone}}$




$V = \pi r^2 H' - V_c = \pi \times 1.07^2 \times 10 - 4.80 =$

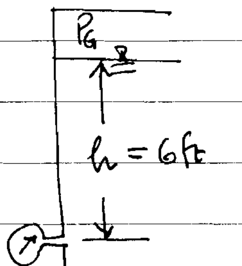
$V = 35.96\ ft^3 - 4.80 ft^3 = 31.16\ ft^3$

$\boxed{\text{WEIGHT OF OIL}}$

$W' = s\gamma_w V = 0.8 \times 62.4 \frac{lb}{ft^3} \times 31.16 ft^3$

$\underline{W' = 1555.91\ lb}$

$\boxed{\text{GAS PRESSURE}}$




$P_B = 0.5\ psi = 0.5 \times 144 \frac{lb}{ft^2}$

$P_B = 72\ lb/ft^2$

$P_G + s\gamma_w h = P_B$

$P_G = P_B - s\gamma_w h$

$P_G = 72 \frac{lb}{ft^2} - 0.8 \times 62.4 \frac{lb}{ft^3} \times 6ft$

$P_G = 72 \frac{lb}{ft^2} - 299.52 \frac{lb}{ft^2}$

$\underline{P_G = -227.52\ lb/ft^2}$

VACUUM

$\boxed{\text{GAS OVERLOAD IS ACTUALLY SUCTION}}$

$F_G = P_G \cdot A = P_G \cdot \pi r^2 = (-227.52 \frac{lb}{ft^2}) \pi \times (1.07 ft)^2$

$\underline{F_G = -818.34\ lb}$

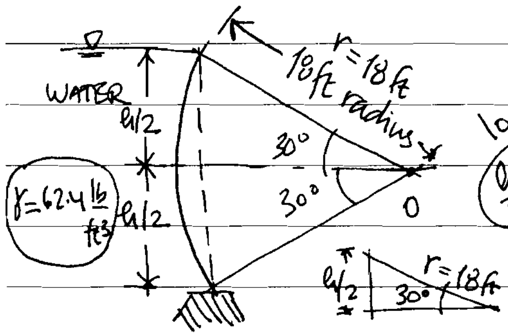
$\boxed{\text{NET VERTICAL FORCE (DOWNWARDS)}}$

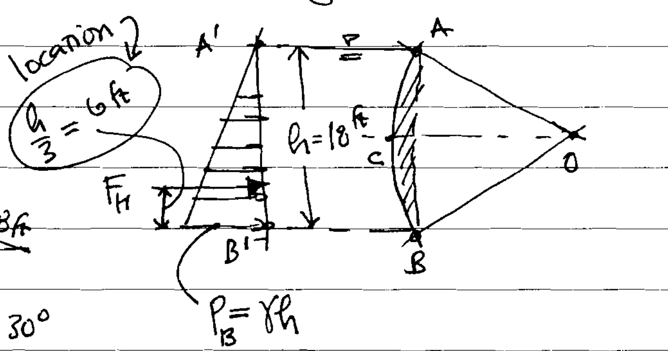
$F_V = W' + F_G = 1555.91 lb + (-818.34 lb)$

$\boxed{F_V = 737.57\ lb}$

3.20. (a) Find the horizontal and vertical forces per foot of width acting on the Tainter gate in Fig. P3.20. (b) Locate the horizontal force and indicate the line of action of the vertical force without actually computing its location. (c) Locate the vertical force. (Hint: Consider the resultant).

(a) calculating forces, width, w = 1 ft




$\sin 30^\circ = \frac{h/2}{r} \Rightarrow h = 2r\sin 30^\circ$

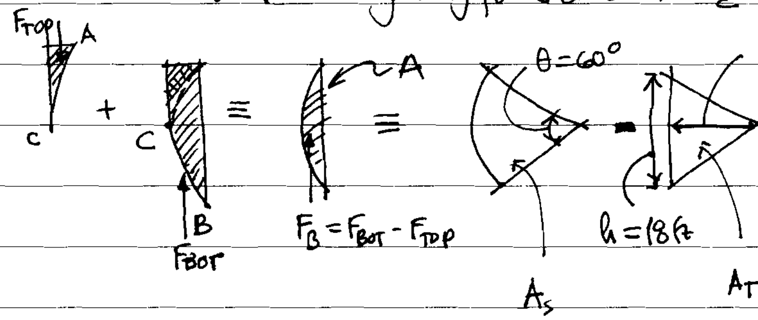
$h = 2 \times 18\text{ ft} \times \sin 30^\circ$

$\boxed{h = 18\text{ ft}}$

$F_H = \frac{1}{2}(\gamma h)(h)(w) = \frac{1}{2}\gamma h^2 w$

$\rightarrow \boxed{F_H = \frac{1}{2} \times 62.4 \times 18^2 \times 1 = 10108.8\ lb}$

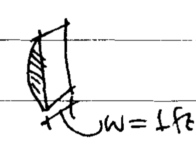
VERTICAL FORCE, $F_V$ = buoyancy force on sector AB = $\gamma \cdot \forall$




$18\cos 30^\circ = 15.59$ ft

$A_T = \frac{1}{2} \times 18 \times 15.59$

$A_T = 140.31\ ft^2$



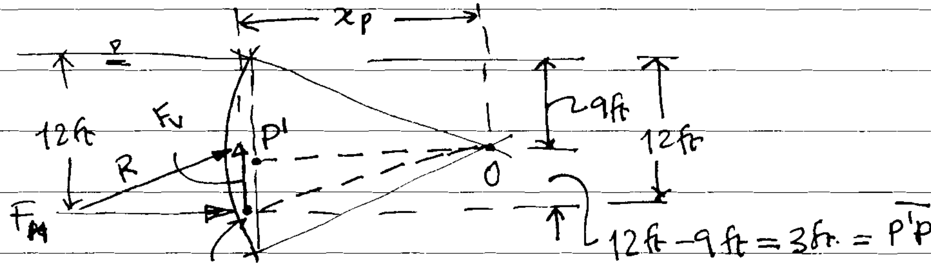

AREA OF SECTOR, $\frac{A_s}{\pi r^2} = \frac{\theta}{360^\circ} \Rightarrow A_s = \frac{\theta}{360^\circ} \cdot \pi r^2 = \frac{60^\circ}{360^\circ} \cdot \pi \cdot 18^2 = 169.64\ ft^2$

$A = A_s - A_T = 169.64 - 140.31 = 29.34\ ft^2$, $\forall = A \times w = 29.34 \times 1 = 29.34\ ft^3$

$\uparrow \boxed{F_V = \gamma \forall = 62.4 \times 29.34 = 1830.82\ lb}$

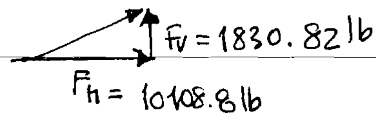
(b) $F_H$ is 6 ft from bottom of gate, $F_V$ goes through center of gravity of ACB

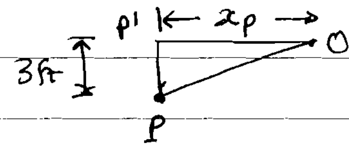
(c) [NOT SO INTUITIVE] → RESULTANT GOES THROUGH POINT O because all forces are normal to the circular gate (see diagram in the following page)




P = CENTER OF PRESSURE

similar triangles




$$\frac{x_p}{F_H} = \frac{3\text{ ft}}{F_V}$$

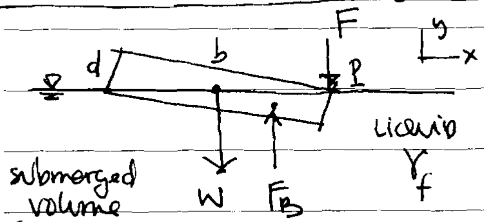
$$x_p = 3\text{ ft} \times \frac{F_H}{F_V} = 3\text{ ft} \times \frac{10108.8\text{ lb}}{1830.82\text{ lb}}$$

$$\boxed{x_p = 16.56\text{ ft}}$$ TO THE LEFT OF POINT O

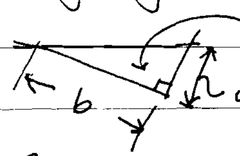
3.29. A rectangular block of uniform material and length $L = 800$ mm, width $b = 300$ mm, and depth $d = 50$ mm, is floating in a liquid. It assumes the position shown in Fig. P3.28 when a uniform vertical load of 20 N/m is applied at P. (a) Find the weight of the block. (b) If the load is suddenly removed, what is the righting moment before the block starts to move? (Hint: Refer also to Fig. 3.19)

(a) $\omega = 20$ N/m, on P

$F = \omega L = (20\,N/m)(0.8\,m) = 16\,N$




Buoyancy force, $F_B$




$A_s = \frac{1}{2} bd = \frac{1}{2} \times 0.3^m \times 0.05^m$

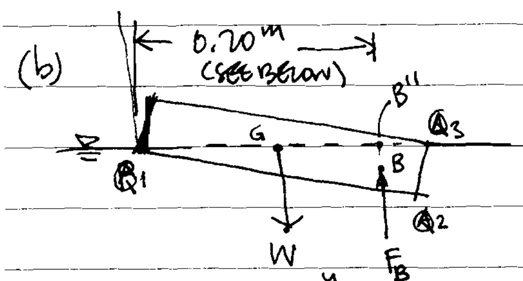
$A_s = 0.0075\,m^2$

$\forall_s = A_s \cdot L = 0.0075\,m^2 \times 0.8\,m = 0.006\,m^3$, $F_B = \gamma_f \cdot \forall = 0.006\gamma$

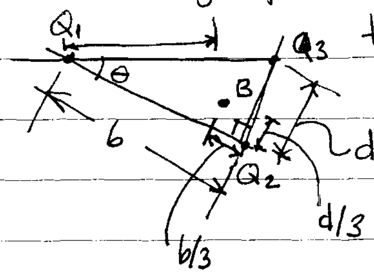
EQUILIBRIUM: $\overset{\oplus}{\uparrow} \Sigma F_y = 0$, $-W + F_B - F = 0 \Rightarrow -W + 0.006\gamma - 16 = 0$

$$\boxed{W = 0.006\gamma - 16}$$

(b)

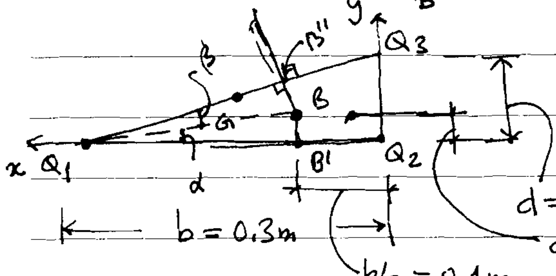



Point B is the centroid of the triangle in the submerged part




$\tan\theta = \frac{d}{b} = \frac{50}{300}$

$\theta = 9.46°$




$\overline{Q_1Q_3} = \sqrt{0.3^2 + 0.05^2} = 0.304^m$

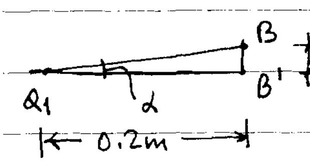
Arm of moment

$\overline{GB''} = \overline{GQ_3} - \overline{B''Q_3}$

$= \frac{1}{2}\overline{Q_1Q_3} - \overline{B''Q_3}$

$= \frac{1}{2} \times 0.304 - 0.104$

$= 0.048^m$




$\tan\alpha = \frac{0.0166}{0.20}$

$\alpha = 4.74°$

$\alpha + \beta = \theta \Rightarrow \beta = \theta - \alpha = 9.46° - 4.74° = 4.72°$

$Q_1(0.3, 0.0)$
$B(0.1, 0.0166)$
$Q_3(0, 0.05)$

$\overline{Q_1B} = \sqrt{(0.3-0.1)^2 + (0.0 - 0.0166)^2}$

$\overline{Q_1B} = 0.20^m$

$\overline{Q_1B''} = \overline{Q_1B} \cdot \cos\beta = 0.20 \times \cos 4.72° = 0.20\,m$

$\overline{B''Q_3} = \overline{Q_1Q_3} - \overline{Q_1B''} = 0.304^m - 0.20^m = 0.104^m$

Body rotates about G, thus

$$\boxed{\begin{aligned} M &= \overline{GB''} \times F_B = 0.048 \times 0.006\gamma \\ &= 0.000288\,\gamma \end{aligned}}$$