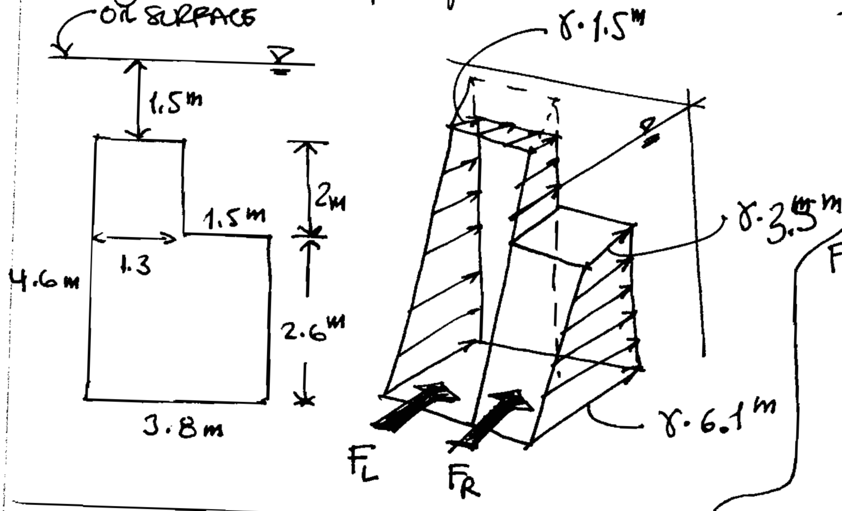
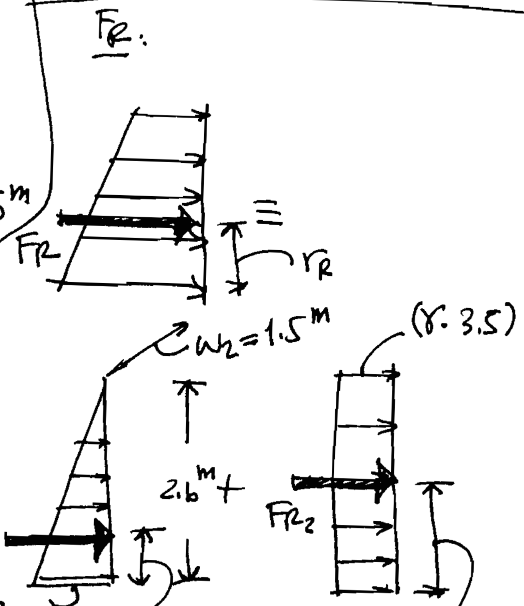
3.14. The Utah-shaped plate shown in Fig. P3.14 is submerged in oil ($s = 0.94$) and lies in a vertical plane. Find the magnitude and locations of the hydrostatic force acting on one side of the plane.



F_R:



$r_{R_1} = \frac{1}{3} \times (2.6^m) = 0.87^m$

$r_{R_2} = \frac{1}{2}(2.6^m) = 1.3^m$

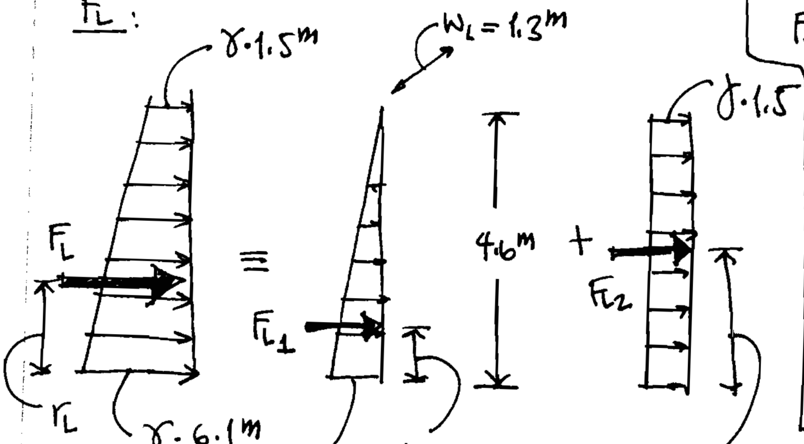
$F_{R_1} = \frac{1}{2}(\gamma \times 2.6) \times (2.6) \times (1.5) = 5.07\gamma$

$F_{R_2} = (\gamma \times 3.5)(2.6) \times (1.5) = 13.65\gamma$

$F_R = F_{R_1} + F_{R_2} = 5.07\gamma + 13.65\gamma$

$F_R = 18.72\gamma$

F_L:



$r_{L_1} = \frac{1}{3}(4.6^m) = 1.53^m$

$r_{L_2} = \frac{1}{2}(4.6^m) = 2.3^m$

$F_{L_1} = \frac{1}{2}(\gamma \times 4.6)(4.6) \times 1.3 = 13.754\gamma$

$F_{L_2} = (\gamma \times 1.5)(4.6) \times 1.3 = 8.97\gamma$

$F_L = F_{L_1} + F_{L_2} = 13.754\gamma + 8.97\gamma = 22.724\gamma$

C.P.L: $r_L \cdot F_L = r_{L_1} \cdot F_{L_1} + r_{L_2} \cdot F_{L_2}$

$r_L \cdot 22.724\cancel{\gamma} = 1.53 \times 13.754\cancel{\gamma} + 2.3 \times 8.97\cancel{\gamma}$

$r_L = \frac{41.675}{22.724} = 1.83^m$

CPR: $r_R F_R = r_{R_1} F_{R_1} + r_{R_2} F_{R_2}$

$r_R \cdot 18.72\cancel{\gamma} = 0.87 \times 5.07\cancel{\gamma} + 1.3 \times 13.65\cancel{\gamma}$

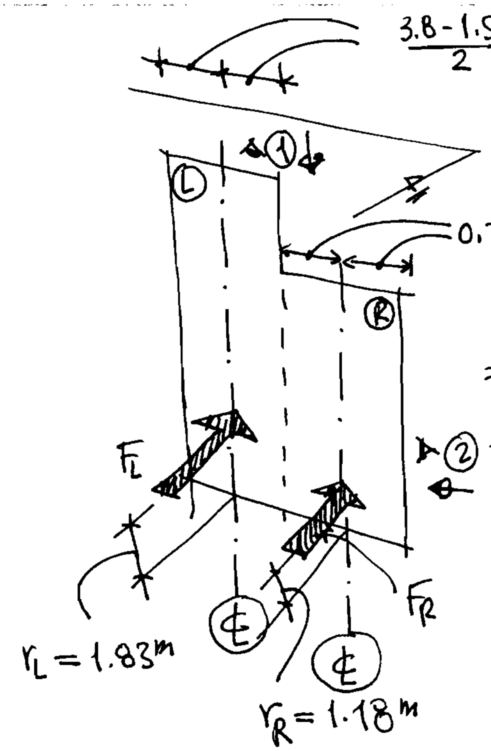
$r_R = \frac{22.156}{18.72} = 1.18^m$

continues in next page

$\frac{3.8-1.5}{2} = 1.15^{m}$ each

$F_L = 22.72\gamma$

$F_R = 18.72\gamma$



$F = F_L + F_R =$

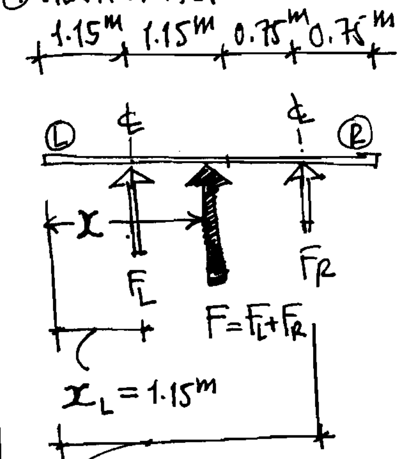
$= 22.72\gamma + 18.72\gamma =$

$= 41.44\gamma$

$$\boxed{\begin{array}{l} \text{with } s = 0.94 \text{ and} \\ \gamma_w = 9810\,N/m^3 \\ \gamma = s\gamma_w = 0.94 \times 9810 \\ \gamma = 9221.4\,\frac{N}{m^3} \end{array}}$$

$$\boxed{\begin{array}{l} F = 41.44\gamma \\ \ \ = 41.44 \times 9221.4 \\ \ \ = 381765.96^{N} \\ \ \ = 381.77\,kN \\ \ \ = 0.38\,MN \end{array}}$$

1 VIEW FROM TOP



$x_R = 2 \times 1.15^{m} + 0.75^{m} = 3.05^{m}$

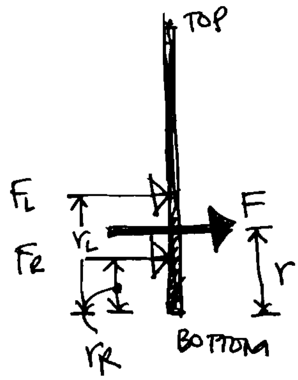
To find x for total force

$x \cdot F = x_L \cdot F_L + x_R \cdot F_R$

$x \cdot 41.44\cancel{\gamma} = 1.15 \times 22.72\cancel{\gamma} + 3.05 \times 18.72\cancel{\gamma}$

$$\boxed{x = \frac{91.648}{41.44} = 2.21^{m}}$$

2 VIEW FROM SIDE

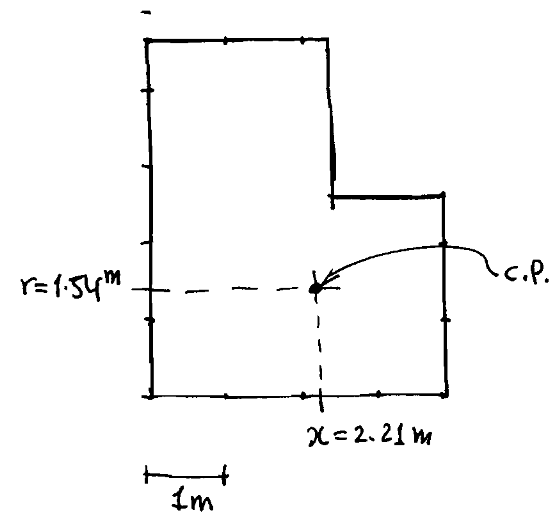


⇒ TO FIND r for the total force

$r \cdot F = r_L \cdot F_L + r_R \cdot F_R$

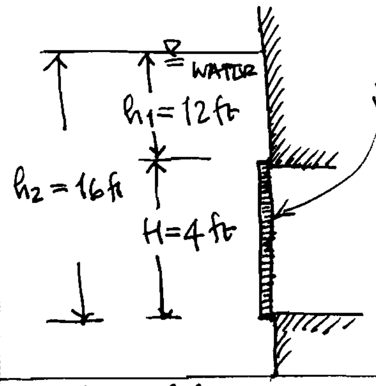
$r \cdot 41.44\cancel{\gamma} = 1.83 \times 22.72\cancel{\gamma} + 1.18 \times 18.72\cancel{\gamma}$

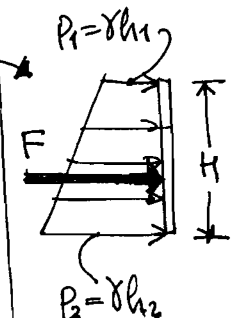
$$\boxed{r = \frac{63.67}{41.44} = 1.54^{m}}$$



3.15. The common type of irrigation headgate shown in Fig. P3.15 is a plate that slides over the opening to a culvert. The coefficient of friction between the gate and its sliding ways is 0.6. Find the force required to slide open this 600-lb gate if it is set (a) vertically; (b) on a 2:1 slope (n=2) as is common.

(a) SET VERTICALLY





$F = \text{volume of pressure prism} = \frac{1}{2}(P_1+P_2)\cdot H \cdot w$

$\boxed{\text{HYDROSTATIC FORCE CALCULATION}}$

$= \gamma\left(\frac{h_1+h_2}{2}\right)H\cdot w \qquad \gamma = 62.4\ \text{lb/ft}^3$

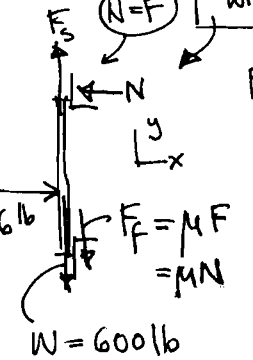
$F = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{12\text{ft}+16\text{ft}}{2}\right)\cdot 4\text{ft}\cdot 4\text{ft}$

$F = 13977.6\ \text{lb}$

Analysis of forces including friction:

normal force, $F = 13977.6$ lb

coefficient of friction, $\mu = 0.6$



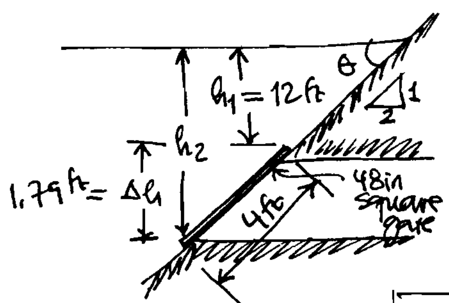
F_s = minimum force required to slide open (upwards)

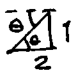
$+\uparrow \Sigma F_y = 0$ (zero acceleration, sliding at constant velocity)

$F_s - W - \mu F = 0 \Rightarrow F_s = W + \mu F$

$\boxed{F_s = 600^{\text{lb}} + 0.6\times 13977^{\text{lb}} = 8986.56}$

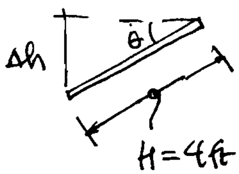
(b) SET A SLOPE 2:1





$\tan\theta = 1/2 \Rightarrow \theta = \tan^{-1}(1/2) = 26.57°$

GATE:



$\sin\theta = \frac{\Delta h}{H} \Rightarrow \Delta h = H\sin\theta = 4^{\text{ft}}\sin 26.57°$

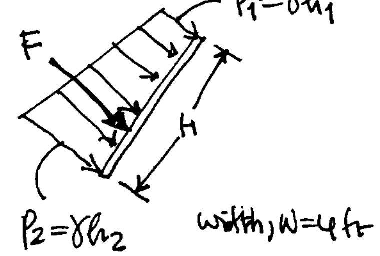
$\underline{\Delta h = 1.79\ \text{ft}}$

$h_2 = h_1 + \Delta h = 12^{\text{ft}} + 1.79\text{ft} = 13.79\ \text{ft}.$

hydrostatic force calculation

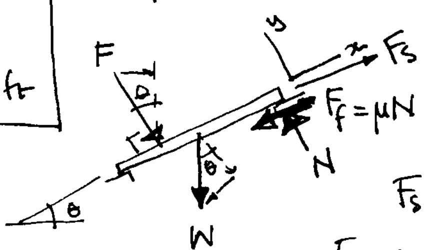
$F = \text{volume of pressure prism} = \gamma\left(\frac{h_1+h_2}{2}\right)H\cdot w = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{12^{\text{ft}} + 13.79^{\text{ft}}}{2}\right)(4\text{ft})(4\text{ft})$

$F = 12874.37^{\text{lb}}$



coefficient of friction, $\mu = 0.60$

Analysis of forces



$+\Sigma F_y = 0, \quad -F - W\cos\theta + N = 0$

$N = F + W\cos\theta$

$= 12874.37 + 600\cos 26.57°$

$= 13411.00\ \text{lb}$

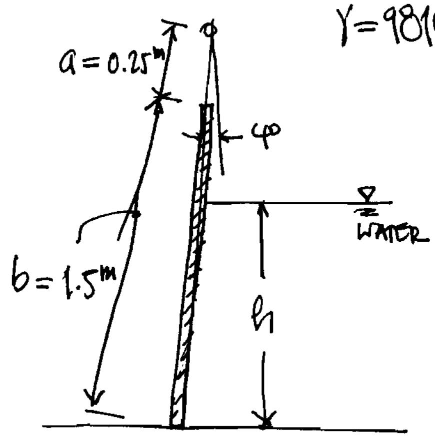
$+\nearrow \Sigma F_x = 0, \quad F_s - F_f - W\sin\theta = 0$

$F_s = F_f + W\sin\theta = \mu N + W\sin\theta$

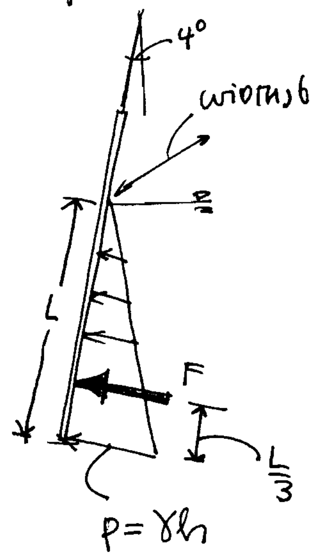
$F_s = 0.6\times 13411.00\text{lb} + 600\sin 26.57°$

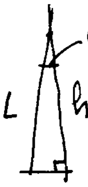
$\boxed{F_s = 8314.97^{\text{lb}} \approx 8315^{\text{lb}}}$

3.16. In the drainage of irrigated lands it is frequently desirable to install automatic flap gates to prevent a flood from backing up into the lateral drains from a river. Suppose a square flap gate, of side $b = 1.5\,m$ and weight 8 KN, is hinged 1 m above its center ($a = 0.25\,m$), as shown in the figure, and the face is sloped 4° from the vertical. To what depth will water rise behind the gate before it will open?



$\gamma = 9810\,N/m^3$





$\cos 4° = \frac{h}{L}$

$L = \frac{h}{\cos 4°}$

$F =$ volume of pressure prism $= \frac{1}{2} p \cdot L \cdot b$

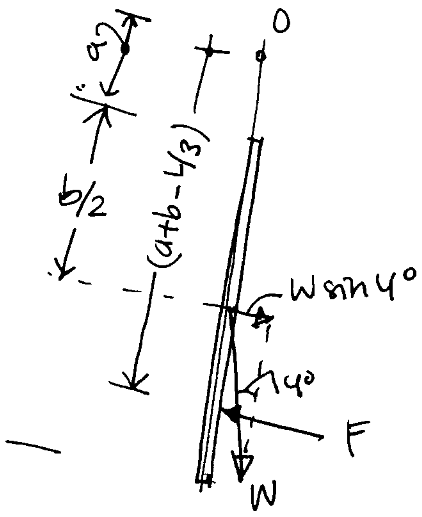
$F = \frac{1}{2}\gamma h L b$

$F = \frac{1}{2}\gamma h \frac{h}{\cos 4°} b$

$$\boxed{F = \frac{\gamma h^2 b}{2\cos 4°}}$$

$\frac{L}{3} = \frac{h}{3\cos 4°}$

FORCES ON GATE



FORCES THAT PRODUCE A MOMENT ABOUT O:

FORCE	ARM
$W \sin 4°$	$a + b/2$
$F = \frac{\gamma h^2 b}{2\cos 4°}$	$a + b - \frac{h}{3\cos 4°}$

The minimum h required to make the gate rise is such that $\circlearrowleft\!+ \sum M_O = 0$

$$-W\sin 4° \times \left(a + \frac{b}{2}\right) + \frac{\gamma h^2 b}{2\cos 4°} \times \left(a + b - \frac{h}{3\cos 4°}\right) = 0$$

$\Rightarrow$ ~~quadratic~~ cubic equation: $\underbrace{-\frac{\gamma b}{6\cos^2 4°}}_{\alpha_1} h^3 + \underbrace{\frac{(a+b)\gamma b}{2\cos 4°}}_{\alpha_2} h^2 - \underbrace{W \sin 4°\left(a + \frac{b}{2}\right)}_{\alpha_3} = 0$

$$\boxed{-\alpha^3 h^3 + \alpha_2 h^2 - \alpha_3 = 0}$$

$\alpha_1 = \frac{\gamma b}{6\cos^2 4°} = \frac{9810 \times 1.5}{6\cos^2 4°} = 2464.49$

$\alpha_2 = \frac{(a+b)\gamma b}{2\cos 4°} = \frac{(0.25+1.5) \times 9810 \times 1.5}{2\cos 4°} = 12907.07$

$\alpha_3 = W\sin 4°\left(a + \frac{b}{2}\right) = 8000 \times \sin 4° \times \left(0.25 + \frac{1.5}{2}\right) = 558.05$

$-2464.49\, h^3 + 12907.07\, h^2 - 558.05 = 0$

solutions: $h = 0.21\,m$

$\boxed{h = 5.22\,m}$

$h = -0.20\,m$