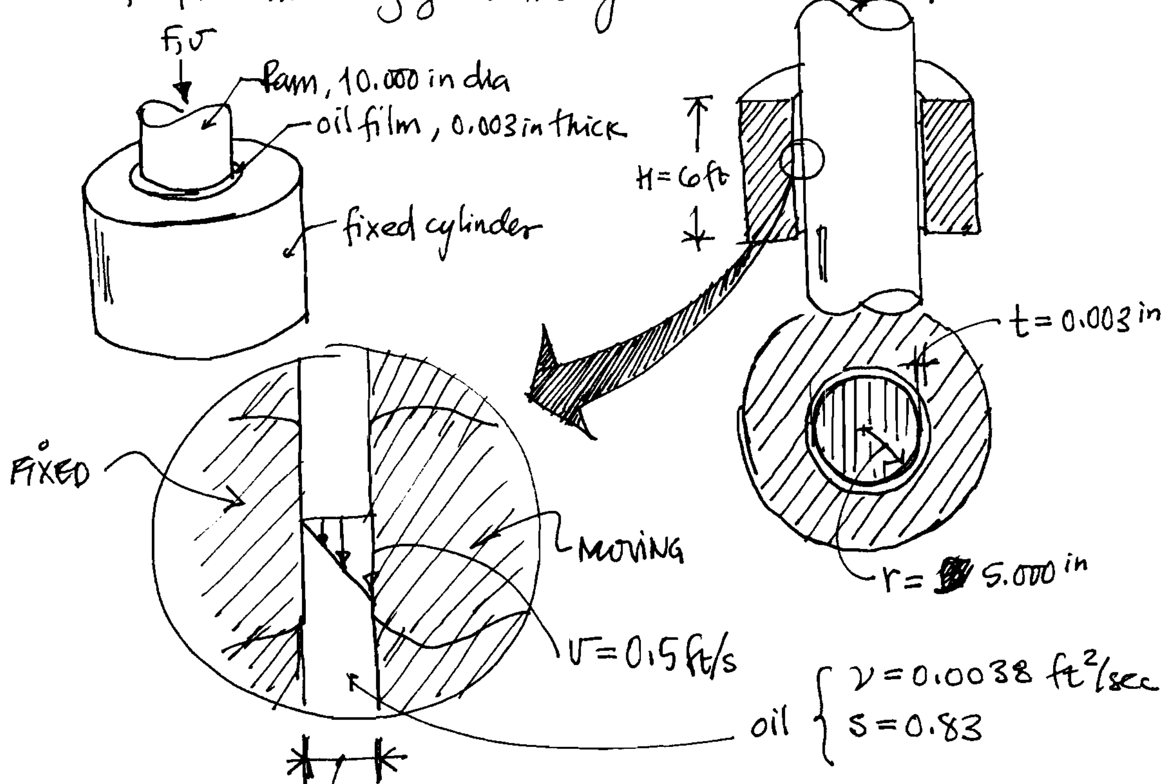
2.21
2.25
2.29

2.21. A hydraulic lift of the type commonly used for greasing automobiles consists of a 10.000-in-diameter ram that slides in a 10.006-in-diameter (Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.0038 ft²/sec and specific gravity of 0.83. If the rate of travel of the ram υ is 0.5 fps, find the frictional resistance, F when 6 ft of the ram is engaged in the cylinder.



$t = 0.003^{in} = \frac{0.003}{12} ft = 0.00025 ft$

$\gamma_w = 62.4\ lb/ft^3$
$g = 32.2 ft/s^2$

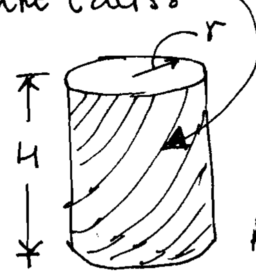
With $s = 0.83$, $\rho = s\rho_w = s\frac{\gamma_w}{g} = 0.83 \times \frac{62.4 \times slug \times \cancel{ft/s^2} \times 1/ft^3}{32.2\ \cancel{ft/s^2}} = 1.608 \frac{slug}{ft^3}$

$\mu = \rho\nu = \left(1.608 \frac{slug}{ft^3}\right)\left(0.0038 \frac{ft^2}{sec}\right) = 6.11 \times 10^{-3} \frac{slug}{ft \cdot sec}$

NOTE: $1 \frac{slug}{ft \cdot sec} = \frac{1\ slug \cdot ft/sec^2}{ft \cdot sec \cdot ft/sec^2} = 1 \frac{lb \cdot sec}{ft^2} \Rightarrow \mu = 6.11 \times 10^{-3} \frac{lb \cdot sec}{ft^2}$

shear stress in gap, $\tau = \mu \frac{\upsilon}{t} = \left(6.11 \times 10^{-3} \frac{lb \cdot sec}{ft^2}\right) \times \frac{0.5 ft/sec}{0.00025 ft} = 12.22 \frac{lb}{ft^2}$

AREA WHERE τ ACTS:

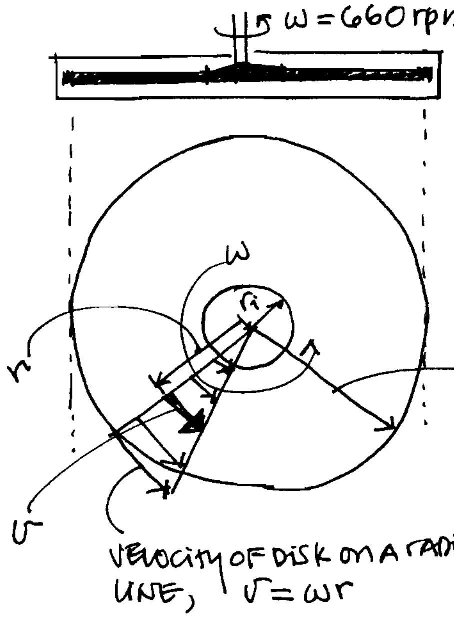


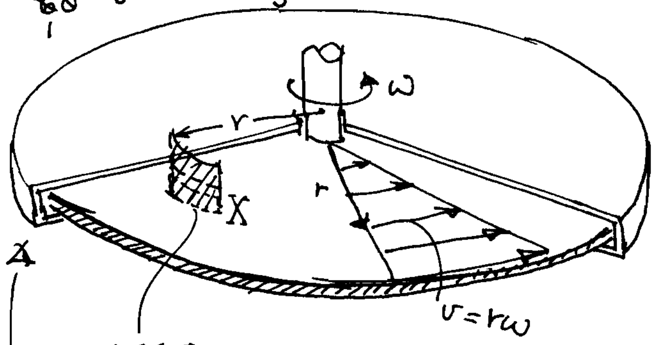
$A = 2\pi r H = 2\pi\left(\frac{5}{12}\right)(6) ft^2 = 5\pi\ ft^2 = 15.71\ ft^2$

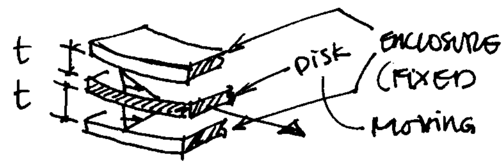
FORCE, $\boxed{F = \tau \cdot A = \left(12.22 \frac{lb}{ft^2}\right)(15.71 ft^2) = 191.98^{lb} \approx 192^{lb}}$

2.25. A disk spins within an oil-filled enclosure, having 2.4-mm clearance from flat surfaces each side of the disk. The disk surface extends from radius 12 to 86 mm. What torque is required to drive the disk at 660 rpm if the oil's absolute viscosity is 0.12 N·s/m²?

$$\omega = 660\,rpm = \cancel{660}^{11} \times \frac{2\pi}{\cancel{60}_{1}}\,\frac{rad}{s} = 22\pi\,\frac{rad}{s} = 69.12\,\frac{rad}{s}$$

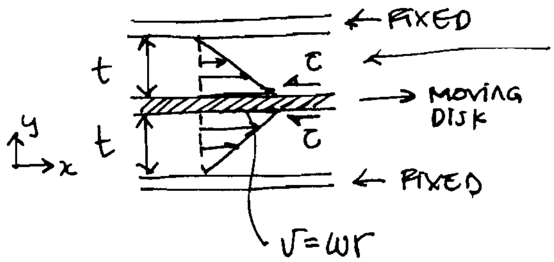




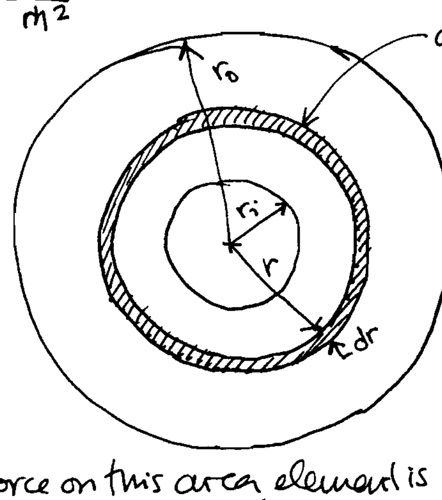


$t = 2.4\,mm = 2.4 \times 10^{-3}\,m$

DETAIL OF VELOCITY DISTRIBUTION



$\mu = 0.12\,\frac{N\text{-}s}{m^2}$



$r_i = 12 \times 10^{-3}\,m$

$r_o = 86 \times 10^{-3}\,m$

in each gap, $\frac{dv}{dy} = \frac{v}{t} = \frac{\omega r}{t}$

$\tau = \mu \frac{dv}{dy} = \frac{\mu \omega r}{t}$

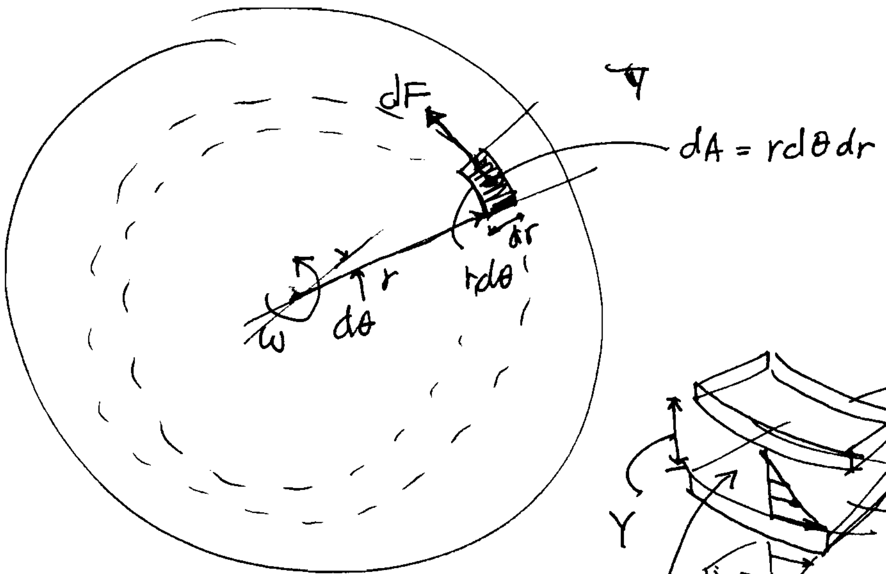
a function of r, thus it applies to the area element of radius r and thickness dr shown in the figure

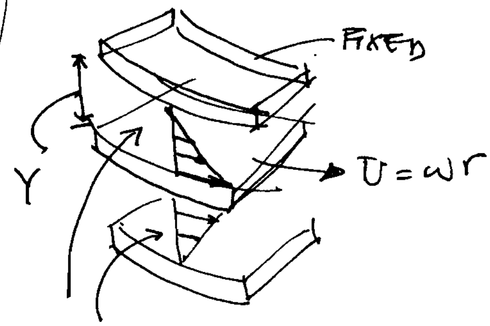
The force on this area element is $dF = \tau\,dA$ and the torque is $dT' = r\tau\,dA$; since there are two faces to the disk, the total torque on dA is $dT = 2\,dT' = 2r\tau\,dA = 2r\frac{\mu\omega r}{t}\,2\pi r\,dr$

$dT = 4\pi\frac{\mu\omega}{t}r^3\,dr$. The total torque is $T = \int dT = \frac{4\pi\mu\omega}{t}\int_{r_i}^{r_o} r^3\,dr = \frac{4\pi\mu\omega}{t}\frac{r^4}{4}\Big|_{r_i}^{r_o}$

$$T = \frac{\pi\mu\omega}{t}(r_o^4 - r_i^4) = \frac{\pi \times (0.12\,N\cdot s/m^2)(69.12\,rad/s)}{2.4 \times 10^{-3}\,m}\left((86 \times 10^{-3})^4 - (12 \times 10^{-3})^4\right)m^4$$

$$\boxed{T = 0.0000546\,N\cdot m = 0.0546\,mN\cdot m = 54.6\,\mu N\cdot m}$$





$$\tau = \mu \frac{U}{Y} = \frac{\mu \omega r}{Y}$$

$$dF = \tau dA =$$

$$= \tau (r d\theta dr)$$

$$= \frac{\mu \omega r^2}{Y} d\theta dr$$

$$dT = r \cdot dF = r\left(\frac{\mu \omega r^2}{Y} d\theta dr\right)$$

$$= \frac{\mu \omega r^3}{Y} d\theta dr$$

$$dT_r = \frac{\mu \omega r^3}{Y} \cdot 2\pi \cdot dr$$

↑ on constant r

$$T = 2\int dT_r = 2\int_{r_i}^{r_o} \frac{\mu \omega r^3}{Y} 2\pi dr = \frac{4\pi \mu \omega}{Y} \int_{r_i}^{r_o} r^3 dr$$

2.29. Pure water at 50°F stands in a glass tube of 0.04-in diameter at a height of 6.78 in. Compute the true static height.

From Table A.1, for $T = 50°F$, water → $\gamma = 62.41\ lb/ft^3$, $\sigma = 0.00509\ lb/ft$

For clean glass, $\theta = 0°$. Equation (2.12) gives the capillary rise, $r = 0.02 in$

$$= \frac{0.02}{12} ft$$

$$= 1.67 \times 10^{-3} ft$$

$$h_c = \frac{2\sigma\cos\theta}{\gamma r} = \frac{2 \times 0.00509\ lb/ft \times \cos 0°}{62.41\ lb/ft^3 \times 1.67\times10^{-3} ft} = 0.0976\ ft$$

$h_c = 0.0976 \times 12^{in} = 1.172\ in$

h_s = actual static head

h = read head

$h = h_s + h_c \Rightarrow h_s = h - h_c = 6.78^{in} - 1.17^{in}$

$$\boxed{h = 5.61\ in}$$