Solutions using matrices

7.30. $f(h, r, \sigma, \gamma)$ or $f(h, r, \sigma, \rho, g) \Rightarrow n = 5,\ k = 3 \Rightarrow n - k = 2$ Π numbers

repeating variables: $r, \sigma,$

$$\Pi_1 = r^{x_1} \sigma^{y_1} g^{z_1} \rho$$

$$\Pi_2 = r^{x_2} \sigma^{y_2} g^{z_2} h$$

Dimension

$[h] = L$

$[r] = L$

$[\sigma] = FL^{-1} = MT^{-2}$

$[\rho] = ML^{-3}$

$[g] = LT^{-2}$

	repeating			non-repeating	
	r	σ	g	ρ	h
M	0	1	0	−1	0
L	1	0	1	3	−1
T	0	−2	−2	0	0
	$\underline{\underline{A}}$			$\underline{\underline{B}}$	

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix} \qquad \underline{\underline{B}} = \begin{bmatrix} -1 & 0 \\ 3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{X}} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ z_1 & z_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix}, \quad \Pi_1 = r^2 \sigma^{-1} g \rho = \frac{\rho g r^2}{\sigma}$$

$$\Pi_2 = r^{-1} \sigma^0 g^0 h = \frac{h}{r}$$

$$\Pi_2 = f(\Pi_1^{-1})$$

$$\boxed{\frac{h}{r} = f\left(\frac{\sigma}{\rho g r^2}\right)}$$

7.31. $f(F_D, L, V, \rho, \mu, E)$

$[F_D] = F,\ [L] = L,\ [V] = LT^{-1},\ [\rho] = FT^2L^{-4}$

$[\mu] = FTL^{-2},\ [E] = FL^{-2}$

$n = 6,\ m = 3 \Rightarrow 6-3 = 3\ \Pi$ terms.

repeating: V, L, ρ

$$\Pi_1 = V^{x_1} L^{y_1} \rho^{z_1} F_D$$
$$\Pi_2 = V^{x_2} L^{y_2} \rho^{z_2} \mu$$
$$\Pi_3 = V^{x_3} L^{y_3} \rho^{z_3} E$$

repeat (V, L, ρ) — non-repeat (F_D, μ, E)

	V	L	ρ	F_D	μ	E
F	0	0	1	-1	-1	-1
L	1	1	-4	0	+2	+2
T	-1	0	2	0	-1	0

$\underline{\underline{A}}$ (V, L, ρ) — $\underline{\underline{B}}$ (F_D, μ, E)

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -4 \\ -1 & 0 & 2 \end{bmatrix} \qquad \underline{\underline{B}} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\underline{\underline{X}} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -2 \\ -2 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\Pi_1 = V^{-2} L^{-2} \rho^{-1} F_D = \frac{F_D}{\rho L^2 V^2}$$

$$\Pi_2 = V^{-1} L^{-1} \rho^{-1} \mu = \frac{\mu}{\rho L V} = \frac{1}{\mathbb{R}}$$ Reynolds

$$\Pi_3 = V^{-2} L^{0} \rho^{-1} E = \frac{E}{\rho V^2} = \frac{E/\rho}{V^2} = \left(\frac{c}{V}\right)^2 = \frac{1}{\mathbb{M}^2}, \quad c = \sqrt{E/\rho} = \text{speed of sound}$$

Mach

$$\Pi_1 = f(\Pi_2^{-1}, \Pi_3^{-2})$$

$$\boxed{\frac{F_D}{\rho L^2 V^2} = f(\mathbb{R}, \mathbb{M})}$$

7.32. $f(P,H,\mu,\rho,\sigma,g)=q$ $\quad [q]=L^2T^{-1}, [H]=L, [P]=L, [\mu]=ML^{-1}T^{-1}$

$n=7, m=3 \Rightarrow 7-3=4$ Π terms $\quad [\rho]=ML^{-3}, [\sigma]=MT^{-2}, [g]=LT^{-2}$

repeating — non-repeating

	q	H	ρ	P	μ	σ	g
M	0	0	1	0	-1	-1	0
L	2	1	-3	-1	+1	0	-1
T	-1	0	0	0	+1	+2	+2

$\underline{A}$ — $\underline{B}$

$$\Pi_1 = q^{x_1}H^{y_1}\rho^{z_1}P$$
$$\Pi_2 = q^{x_2}H^{y_2}\rho^{z_2}\mu$$
$$\Pi_3 = q^{x_3}H^{y_3}\rho^{z_3}\sigma$$
$$\Pi_4 = q^{x_4}H^{y_4}\rho^{z_4}g$$

$$\underline{A} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & -3 \\ -1 & 0 & 0 \end{bmatrix} \qquad \mathbb{B} = \begin{bmatrix} 0 & -1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 & -2 \\ -1 & 0 & 1 & 3 \\ 0 & -1 & -1 & 0 \end{bmatrix}$$

$$\Pi_1 = q^0 H^{-1}\rho^0 P = P/H$$
$$\Pi_2 = q^{-1}H^0\rho^1\mu = \frac{\mu}{\rho q} = \frac{1}{R}$$
$$\Pi_3 = q^{-2}H^1\rho^{-1}\sigma = \frac{\sigma H}{\rho q^2}$$
$$\Pi_4 = q^{-2}H^3\rho^0 g = \frac{gH^3}{q^2}$$