7.30 N *Use dimensional analysis to derive an expression for the height of capillary rise in a glass tube.*

Step 1: $f'(h, r, \sigma, \gamma) = 0$, so no. of variables, $n = 4$. Using FLT:

Step 2: $h = [L]$, $r = [L]$, $\sigma = \left[\frac{F}{L}\right]$, $\gamma = \left[\frac{F}{L^3}\right]$; $m = 2$

Step 3: The 3 variables h, r, and σ can not be formed into a dimensionless group, so the reduction number $k = 3$.

Step 4: No. of Π groups $= n - k = 1$, $\therefore f(\Pi) = 0$.

Steps 5 and 6: For $\Pi = r^a \sigma^b \gamma^c h$: $F^0 L^0 T^0 = (L)^a \left(\frac{F}{L}\right)^b \left(\frac{F}{L^3}\right)^c (L)^1$

F: $0 = b + c$; L: $0 = a - b - 3c + 1$; T: $0 = 0$

Solving: $a = 2c - 1$, $b = -c$. So $\Pi = r^{2c-1}\sigma^{-c}\gamma^c h = (h/r)(\gamma r^2/\sigma)^c$

Step 7: We can write $f(\Pi) = 0$, or $\Pi = C =$ const. Then $h = Cr\left(\frac{\sigma}{\gamma r^2}\right)^c$ ◀

Experimental observation suggests that $c = 1$ (see Sec 2.12), in which case $h = \frac{C\sigma}{\gamma r}$

31 *Derive an expression for the drag on an aircraft flying at supersonic speed.*

N

At supersonic speed the volume modulus of elasticity E_v is also a factor (see Eq 7.10). ∴ we expect both **M** and **R** to govern.

Step 1: $f'(F_D, L, V, \rho, \mu, E_v) = 0$, so no. of variables, $n = 6$. Using FLT:

Step 2: $F_D = [F]$, $L = [L]$, $V = \left[\dfrac{L}{T}\right]$, $\rho = \left[\dfrac{FT^2}{L^4}\right]$, $\mu = \left[\dfrac{FT}{L^2}\right]$, $E_v = \left[\dfrac{F}{L^2}\right]$; $m = 3$

Step 3: The 3 variables V, L, and ρ can <u>not</u> be formed into a dimensionless group,

so the reduction number $k = 3$.

Step 4: No. of Π groups $= n - k = 3$, ∴ $f(\Pi_1, \Pi_2, \Pi_3) = 0$

Steps 5 and 6: Select for the 3 ($=k$) primary (repeating) variables: ρ, L, V

For $\Pi_1 = \rho^a L^b V^c \mu$: $F^0L^0T^0 = \left(\dfrac{FT^2}{L^4}\right)^a (L)^b \left(\dfrac{L}{T}\right)^c \left(\dfrac{FT}{L^2}\right)^1$

F: $0 = a + 1$; L: $0 = -4a + b + c - 2$; T: $0 = 2a - c + 1$

Solving: $a = -1$, $b = -1$, $c = -1$. So $\Pi_1 = \rho^{-1}L^{-1}V^{-1}\mu = \mu/(LV\rho) = \mathbf{R}^{-1}$

For $\Pi_2 = \rho^a L^b V^c F_D$: $F^0L^0T^0 = \left(\dfrac{FT^2}{L^4}\right)^a (L)^b \left(\dfrac{L}{T}\right)^c (F)^1$

F: $0 = a + 1$; L: $0 = -4a + b + c$; T: $0 = 2a - c$

Solving: $a = -1$, $b = -2$, $c = -2$. So $\Pi_2 = \rho^{-1}L^{-2}V^{-2}F_D = F_D/(\rho L^2 V^2)$

For $\Pi_3 = \rho^a L^b V^c E_v$: $F^0L^0T^0 = \left(\dfrac{FT^2}{L^4}\right)^a (L)^b \left(\dfrac{L}{T}\right)^c \left(\dfrac{F}{L^2}\right)^1$

F: $0 = a + 1$; L: $0 = -4a + b + c - 2$; T: $0 = 2a - c$

Solving: $a = -1$, $b = 0$, $c = -2$. So $\Pi_3 = \rho^{-1}L^0V^{-2}E_v = E_v/(\rho V^2) = \mathbf{M}^{-2}$

Step 7: We can write $\Pi_2 = \phi(\Pi_1^{-1}, \Pi_3^{-1/2})$, i.e. $F_D/(\rho L^2 V^2) = \phi(\mathbf{R}, \mathbf{M})$, or $F_D = \rho L^2 V^2 \phi(\mathbf{R}, \mathbf{M})$ ◄

7.32 *Derive an expression for small flow rates over a spillway, in the form of a function including dimensionles quantities. Use dimensional analysis with the following parameters: height of spillway P, head on the spillway H, viscosity of liquid μ, density of liquid ρ, surface tension σ, and acceleration due to gravity g.*

N

Step 1: $f'(q,P,H,g,\mu,\rho,\sigma) = 0$, so no. of variables, $n = 7$. Using MLT:

Step 2: $q = \left[\frac{L^2}{T}\right]$, $P = [L]$, $H = [L]$, $g = \left[\frac{L}{T^2}\right]$, $\mu = \left[\frac{M}{LT}\right]$, $\rho = \left[\frac{M}{L^3}\right]$, $\sigma = \left[\frac{M}{T^2}\right]$; $m = 3$

Step 3: The 3 variables q, H, and ρ can <u>not</u> be formed into a dimensionless group,

so the reduction number $k = 3$.

Step 4: No. of Π groups needed $= n - k = 4$, $\therefore f(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$.

Steps 5 and 6: Select for the 3 ($=k$) primary (repeating) variables: q, H, ρ.

For $\Pi_1 = q^a H^b \rho^c P$: $M^0L^0T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c (L)^1$

M: $0 = c$; L: $0 = 2a + b - 3c + 1$; T: $0 = -a$

Solving: $a = 0, b = -1, c = 0$. So $\Pi_1 = q^0 H^{-1} \rho^0 P = P/H$

For $\Pi_2 = q^a H^b \rho^c g$: $M^0L^0T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{L}{T^2}\right)^1$

M: $0 = c$; L: $0 = 2a + b - 3c + 1$; T: $0 = -a - 2$

Solving: $a = -2$, $b = 3$, $c = 0$. So $\Pi_2 = q^{-2} H^3 g = gH^3/q^2$

For $\Pi_3 = q^a H^b \rho^c \mu$: $M^0L^0T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^1$

M: $0 = c + 1$; L: $0 = 2a + b - 3c - 1$; T: $0 = -a - 1$

Solving: $a = -1, b = 0, c = -1$. So $\Pi_3 = q^{-1} H^0 \rho^{-1} \mu = \mu/q\rho = \mathbf{R}^{-1}$ (as $q = HV$)

For $\Pi_4 = q^a H^b \rho^c \sigma$: $M^0L^0T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{T^2}\right)^1$

M: $0 = c + 1$; L: $0 = 2a + b - 3c$; T: $0 = -a - 2$

Solving: $a = -2$, $b = 1$, $c = -1$. So $\Pi_4 = q^{-2} H \rho^{-1} \sigma = \sigma H/(\rho q^2) = \mathbf{W}^{-2}$ (*as* $q = HV$)

Step 7: We can write $\Pi_2^{-1/2} = \phi(\Pi_1, \Pi_3^{-1}, \Pi_4^{-1/2})$

i.e., $q/(g^{1/2}H^{3/2}) = \phi(H/P, \mathbf{R}, \mathbf{W})$ or $q = g^{1/2}H^{3/2}\phi(H/P, \mathbf{R}, \mathbf{W})$ ◀