7.5 *Models are to be built of the following prototypes: (a) tides; (b) oil flowing through a full pipeline; (c) water jet; (d) flow over the spillway of a dam; (e) a deep submersible vehicle; (f) an airplane flying at speed; (g) a supersonic aircraft; (h) a supersonic missile. For dynamic similarity, indicate which singl dimensionless ratio will govern, and give reasons why.*

N

Governing dimensionless ratios are:

R for parts (*b*), (*e*), and (*f*), because for these the significant forces are inertia and fluid friction due to viscosity (air compressibility is not appreciable at low airplane speeds). ◄

F for parts (*a*), (*c*), and (*d*), because for these the significant forces are inertia and gravity. ◄

M for parts (*g*) and (*h*), because for these compressibility is important. ◄

7.9 *A 500-ft-long ship will operate at a speed of 20 mph in ocean water whose viscosity is 1.2 cP and specific weight is 64 lb/ft³. What should be the kinematic viscosity of the liquid used with a 10-ft-long model of the ship so that both the Reynolds number and the Froude number would be the same? Does such a liquid exist?*

B

$\mathbf{R}_p = \mathbf{R}_m$; $\mathbf{F}_p = \mathbf{F}_m$; $L_r V_r / \nu_r = 1$; $V_r/(g_r L_r)^{1/2} = 1$

To satisfy both R and F, $[LV/\nu]_r = [V/(g/L)^{1/2}]_r$, i.e. $\nu_r = [(gL)^{1/2}L]_r = L_r^{3/2}$ assuming $g_r = 1.0$

$L_r = L_p/L_m = 500/10 = 50$; $\nu_r = \nu_p/\nu_m = (50)^{3/2} = 354$

$\mu_p = 1.2\ \text{cP} = 1.2 \times 10^{-3}\ \text{N·s/m}^2 = 0.0209(1.2 \times 10^{-3})\ \text{lb·sec/ft}^2 = 2.51 \times 10^{-5}\ \text{lb·sec/ft}^2$

Eq 2.11: $\nu_p = \mu_p/\rho = 2.51 \times 10^{-5}/(64/32.2) = 1.261 \times 10^{-5}\ \text{ft}^2/\text{sec}$

$\nu_m = \nu_p/\nu_r = 1.261 \times 10^{-5}/354 = 3.57 \times 10^{-8}\ \text{ft}^2/\text{sec}$ ◄

Fig A.2: There is no such liquid available. ◄

7.10 *Water flows over a spillway at 5000 cfs. For dynamic similarity, what should the model scale be if the flow rate over the model is to be 45 cfs? The force exerted on a certain area of the model is 1.0 lb. What would the force be on the corresponding area of the prototype?*

BG

Gravity and inertia govern, so satisfies Froude's number. Equating these, we get

Eq 7.2: $V_r = V_p/V_m = (L_p/L_m)^{1/2} = L_r^{1/2}$ assuming $g_m = g_p$

$Q_r = (A \times V)_r = (L^2 \times L^{1/2})_r = L_r^{5/2}$; $(5000/45) = L_r^{5/2}$;

$L_r = L_p/L_m = 6.58$; $\lambda = L_m/L_p = 1:6.58$ ◄

$F_r = (\rho V^2 L^2)_r = \rho_r V_r^2 L_r^2 = 1(L_r)L_r^2 = L_r^3$ assuming $\rho_p = \rho_m$. [Alternatively, $F_r = (\rho Q V)_r$.]

$(F_p/1.0) = (6.58)^3 = 285$; $F_p = 1.0(285) = 285\ \text{lb}$ ◄

7.11 *A 1:600 scale model is built to study tides. (a) What length of time in the model corresponds to one day in the prototype? (b) Suppose this model could be tested on the moon where g is one-sixth of that on earth. What then would be the time relationship between the model and prototype?*

N

Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_p = \mathbf{F}_m$ (Eq. 7.9).

(*a*) $[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$; $\therefore V_r = (gL)_r^{1/2}$. Eq. 7.3: $T_r = L_r/V_r = L_r/(gL)_r^{1/2} = \sqrt{L_r/g_r}$

$T_m = T_p/T_r = T_p/\sqrt{L_r/g_r} = T_p\sqrt{g_r/L_r} = T_p\sqrt{(g_p/g_m)/(L_p/L_m)}$

On earth: $T_m = (24 \text{ hr})\sqrt{(1/1)/(600/1)} = 24 \text{ hr}/24.5 = 0.980 \text{ hr}$ ◄

(*b*) On the moon: $T_m = (24 \text{ hr})\sqrt{(6/1)/(600/1)} = 24 \text{ hr}/10 = 2.40 \text{ hr}$ ◄

7.12 *A vertical jet of water issuing upward from a nozzle at a velocity of 44 fps will rise to a height of approximately 30 ft on the earth. To get a water jet to rise to a height of 120 ft on the moon, where the gravity in one-sixth of that on earth, what must the jet velocity be? Neglect atmospheric resistance.*

BG

Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_e = \mathbf{F}_m$.

Eq. 7.1: $L_r = L_e/L_m = 30/120 = 0.25$; $g_r = g_e/g_m = 6$

Eq. 7.9: $[V/(gL)^{1/2}]_e = [V/(gL)^{1/2}]_m$; $\therefore$ $V_e/V_m = \sqrt{(g_e/g_m)(L_e/L_m)} = \sqrt{6(0.25)} = 1.225 = V_r$ (Eq. 7.2)

$V_m = V_e/V_r = 44/1.225 = 35.9 \text{ fps}$ ◄

7.13 *A 3-ft-high sectional model of a spillway is built in a 1-ft-wide laboratory flume. The flow is 0.86 cfs under a head of 0.380 ft. If the model scale is 1:20 and the prototype spillway is 600 ft long, what flow does this represent in the prototype?*

BG

$L_r = L_p/L_m = 20$; $g_r = 1.0$. Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_m = \mathbf{F}_p$ (Eq. 7.9).

$[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$ so $V_r = (gL)_r^{1/2} = \sqrt{g_r L_r} = \sqrt{1(20)} = 4.47$; $A_r = L_r^2 = 20^2 = 400$

$Q_r = A_r V_r = 400(4.47) = 1789$; $Q_m = (0.86 \text{ cfs/ft})(600/20 \text{ ft}) = 25.8 \text{ cfs}$

$Q_p = Q_m Q_r = (25.8 \text{ cfs})1789 = 46{,}200 \text{ cfs}$ ◄