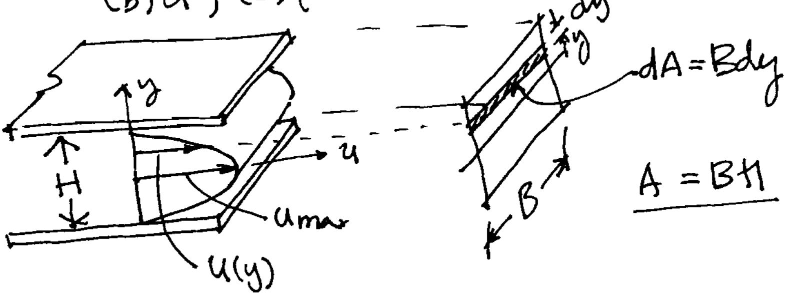
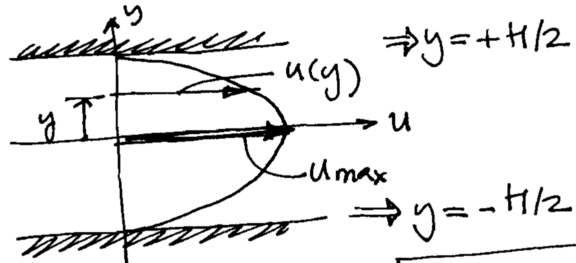
6.1. Two-dimensional laminar flow between two stationary parallel plates (parabolic velocity distribution). Find (a) $V/u_{max} = ?$ (b) α ; (c) β.





parabolic distribution: $\boxed{u(y) = u_{max}\left(1-\left(\frac{y}{H/2}\right)^2\right)}$

check: when $y = 0 \rightarrow u(y) = u_{max}$

when $y = \pm H/2 \rightarrow u(y) = 0$

$$Q = \int_A u\,dA = \int_{-H/2}^{+H/2} u_{max}\left[1-\left(\frac{2y}{H}\right)^2\right]\cdot B\cdot dy = \frac{2HBu_{max}}{3}$$

$$V = \frac{Q}{A} = \frac{\frac{2}{3}HBu_{max}}{BH} = \frac{2}{3}u_{max} \Rightarrow \text{(a)}\ \boxed{\frac{V}{u_{max}} = \frac{2}{3} = 0.666}$$

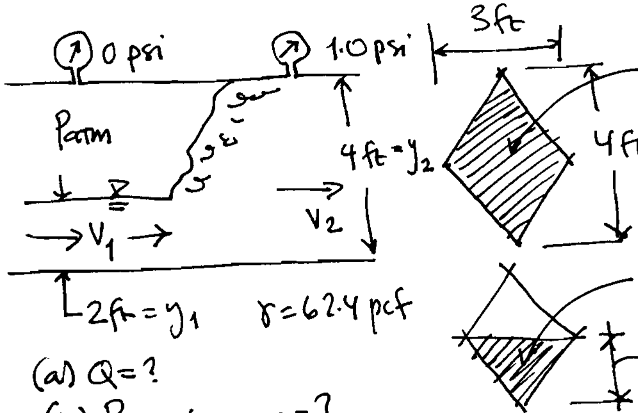
$$\int_A u^2 dA = \int_{-H/2}^{H/2} u_{max}^2\left[1-\left(\frac{2y}{H}\right)^2\right]^2\cdot B\,dy = \frac{8}{15}B\cdot H\cdot u_{max}^2$$

$$\int_A u^3 dA = \int_{-H/2}^{H/2} u_{max}^3\left[1-\left(\frac{2y}{H}\right)^2\right]^3 B\,dy = \frac{16}{35}B\cdot H\cdot u_{max}^3$$

$$\text{(b)}\ \boxed{\alpha = \frac{1}{AV^3}\int_A u^3 dA = \frac{1}{B\cdot H\cdot\left(\frac{2}{3}u_{max}\right)^3}\cdot\frac{16}{35}B\cdot H\cdot u_{max}^3 = \frac{54}{35} = 1.54}$$

$$\text{(c)}\ \boxed{\beta = \frac{1}{AV^2}\int_A u^2 dA = \frac{1}{B\cdot H\cdot\left(\frac{2}{3}u_{max}\right)^2}\cdot\frac{8}{15}BH\cdot u_{max}^2 = \frac{6}{5} = 1.20}$$

6.4. hydraulic jump in close conduit (diamond-shape)



$\gamma = 62.4$ pcf

AREA, $A_2 = \frac{1}{2}(3)(4) =$
$A_2 = 6.0 \text{ ft}^2$

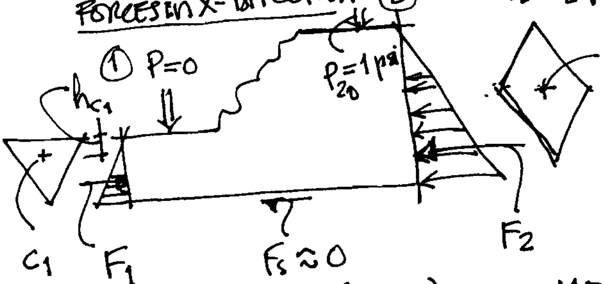
AREA, $A_1 = \frac{1}{2} A_2$
$= 3.0 \text{ ft}^2$

(a) $Q = ?$
(b) Power in jump $= ?$

- Turbulence in jump $\Rightarrow$ can not use energy equation
- use momentum equation $h_{c_1} = \frac{1}{3}(y_1)$, $h_{c_2} = \frac{1}{2} y_2$

$F_1, F_2 =$ hydrostatic forces

FORCES IN X-DIRECTION



$F_1 = (P_1)_c A_1 = \gamma h_{c_1} A_1$

$F_2 = (P_2)_c A_2 = (P_{2_0} + \gamma h_{c_2}) A_2$

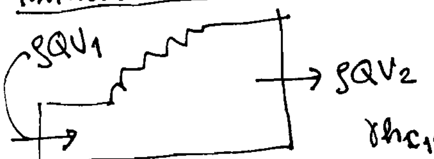
NOTE: SECTION ② is pressurized in the surface

MOMENTUM principle in x-direction

$\sum F_x = \Delta(\rho Q V)_x$

$F_1 - F_2 - F_s = \rho Q (V_2 - V_1)$

MOMENTUM FLOW IN X DIRECTION



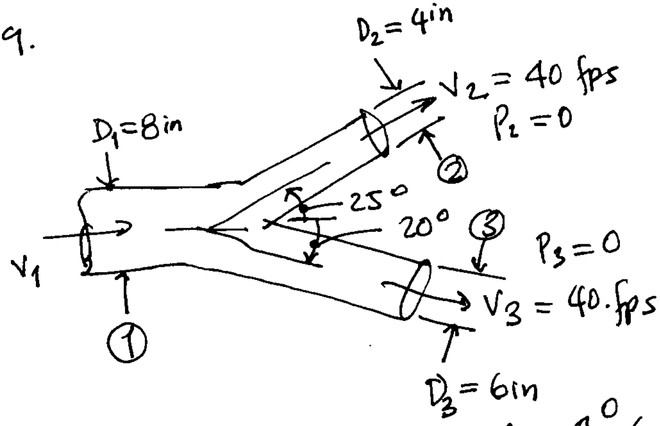
$\gamma h_{c_1} A_1 - (P_{2_0} + \gamma h_{c_2}) A_2 - 0 = \rho Q (V_2 - V_1)$

use $V_2 = \frac{Q}{A_2}$, $V_1 = \frac{Q}{A_1}$ $\Rightarrow$ $\gamma h_{c_1} A_1 - P_{2_0} A_2 - \gamma h_{c_2} A_2 = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1}\right)$

$$Q^2 = \frac{\gamma(h_{c_1} A_1 - h_{c_2} A_2) - P_{2_0} A_2}{\rho\left(\frac{1}{A_2} - \frac{1}{A_1}\right)} = \frac{62.4\left(\frac{2}{3}\times 3 - \frac{4}{2}\times 6\right) - 1\times 144\times 6}{\frac{62.4}{32.2}\left(\frac{1}{6} - \frac{1}{3}\right)}$$

$Q^2 = 4607.07 \text{ cfs}^2 \Rightarrow \boxed{Q = 67.87 \text{ cfs}}$

6.9.



$\gamma = 62.4 \frac{lb}{ft^3}$

HORIZONTAL PLANE

$z_1 = z_2 = z_3 =$ same

- Find Force on DOUBLE NOZZLE
- NEGLECT FRICTION ($h_L = 0$)

ENERGY ① - ②: $\frac{P_1}{\gamma} + \cancel{z_1} + \frac{V_1^2}{2g} + \cancel{h_{L_{1-2}}}^{0} = \cancel{\frac{p_2}{\gamma}}^{0} + \cancel{z_2} + \frac{V_2^2}{2g}$

$\Rightarrow \frac{P_1}{\gamma} = \frac{V_2^2 - V_1^2}{2g}$

continuity: $Q_1 = Q_2 + Q_3 \Rightarrow V_1 \cdot \cancel{\frac{\pi}{4}} D_1^2 = V_2 \cdot \cancel{\frac{\pi}{4}} D_2^2 + V_3 \cancel{\frac{\pi}{4}} D_3^2$

$$V_1 = \frac{V_2 D_2^2 + V_3 D_3^2}{D_1^2} = \frac{40 \times 4^2 + 40 \times 6^2}{8^2} = 32.5 \text{ fps}$$

$$\frac{P_1}{\gamma} = \frac{40^2 - 32.5^2}{2 \times 32.2} = 8.44 \text{ ft} \Rightarrow P_1 = 8.44 \times 62.4 = 526.86 \frac{lb}{ft^2}$$

MOMENTUM:

- FORCES

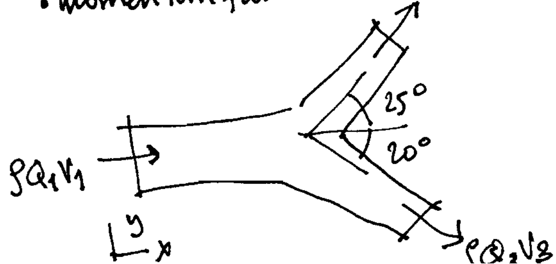
$\Sigma F_x = P_1 A_1 - F_x$

$\Sigma F_y = F_y$

$\Delta(\rho Q V)_x =$ OUT - in

$= \rho Q_2 V_{2x} + \rho Q_3 V_{3x} - \rho Q_1 V_{1x}$

- momentum flows



$\Delta(\rho Q V)_y =$ OUT - IN

$= \rho Q_2 V_{2y} + \rho Q_2 V_{3y} - \rho Q_1 V_{1y}$

Discharges:

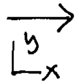
$$Q_1 = V_1 \frac{\pi D_1^2}{4} = \cancel{40}\,32.5 \times \frac{\pi \times (8/12)^2}{4} = 11.34 \text{ cfs}$$

$$Q_2 = V_2 \frac{\pi D_2^2}{4} = 40 \times \frac{\pi \times (4/12)^2}{4} = 3.49 \text{ cfs}$$

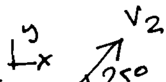
$$Q_3 = V_3 \cdot \frac{\pi D_3^2}{4} = 40 \times \frac{\pi \times (6/12)^2}{4} = 7.85 \text{ cfs}$$

check, $\underline{Q_2 + Q_3 = Q_1}$

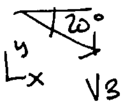
velocity components:



$V_1 : \begin{cases} V_{1x} = V_1 = 32.5 \text{ fps} \\ V_{1y} = 0 \end{cases}$



$: \begin{cases} V_{2x} = V_2 \cos 25^\circ = 36.25 \text{ fps} \\ V_{2y} = V_2 \sin 25^\circ = 16.90 \text{ fps} \end{cases}$



$: \begin{cases} V_{3x} = V_3 \cos 20^\circ = 37.59 \text{ fps} \\ V_{3y} = -V_3 \sin 20^\circ = -13.68 \text{ fps} \end{cases}$

<u>Components of momentum principle</u>

$$\Sigma F_x = \Delta(\rho Q V)_x \Rightarrow P_1 A_1 - F_x = \rho\,(Q_2 V_{2x} + Q_3 V_{3x} - Q_1 V_{1x})$$

$$F_x = P_1 A_1 - \rho\,(Q_2 V_{2x} + Q_3 V_{3x} - Q_1 V_{1x})$$

$$= 526.86 \times \frac{\pi \times (8/12)^2}{4} - \frac{62.4}{32.2}(3.49 \times 36.25 + 7.85 \times 37.59 - 11.34 \times 32.5)$$

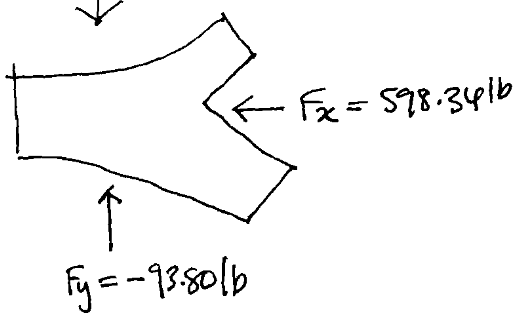
$$\underline{F_x = 598.34 \text{ lb}}$$

$$\Sigma F_y = \Delta(\rho Q V)_y \Rightarrow F_y = \rho(Q_2 V_{2y} + Q_3 V_{3y} - Q_1 V_{1y})$$

$$F_y = \frac{62.4}{32.2}(3.49 \times 16.90 + 7.85 \times (-13.68) - 11.34 \times 0)$$

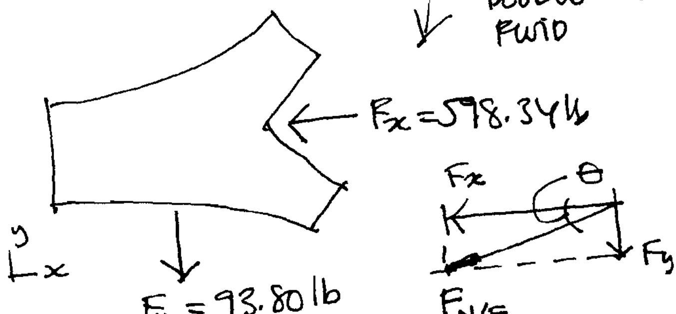
$$\underline{F_y = -93.80 \text{ lb}}$$

mathematical results



TRUE FORCE DIRECTIONS

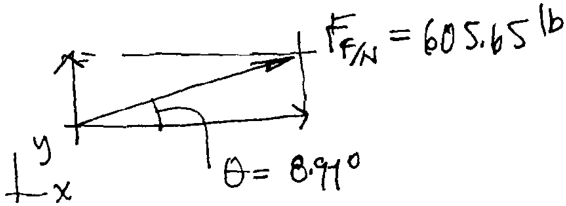
FORCE components of DOUBLE NOZZLE on FLUID



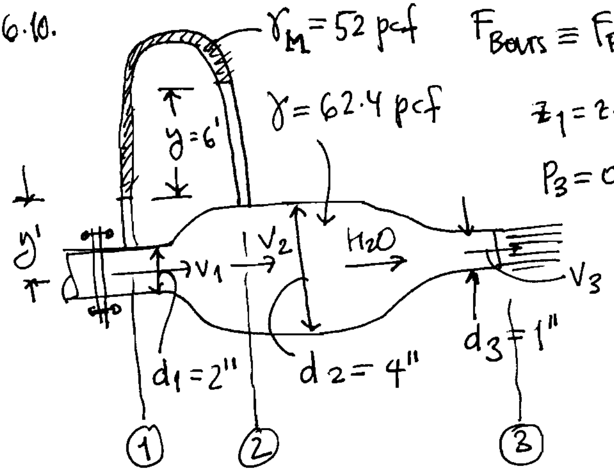
magnitude: $F_{N/F} = \sqrt{F_x^2 + F_y^2} = \sqrt{598.34^2 + 93.80^2} = 605.65$ lb

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{93.80}{598.34}\right) = 8.91°$

FORCE EXERTED BY FLOW ON DOUBLE NOZZLE: same magnitude, but opposite direction as force from nozzle on flow, i.e., $\vec{F}_{F/N} = -\vec{F}_{N/F}$



6.10.



$F_{BOLTS} \equiv F_{FLOW/DEVICE} = -F_{DEVICE/FLOW}$

$z_1 = z_2 = z_3 = \text{constant}$

$P_3 = 0$ (open to atmosphere)

CONTINUITY

$$Q = V_1 \frac{\pi d_1^2}{4} = V_2 \frac{\pi d_2^2}{4} = V_3 \frac{\pi d_3^2}{4}$$

$$\frac{V_1}{V_2} = \left(\frac{d_2}{d_1}\right)^2$$

MANOMETER

$$p_1 - \cancel{\gamma y'} - \gamma_m y + \gamma(y + \cancel{y'}) = p_2 \Rightarrow$$

$$p_2 = p_1 + y(\gamma - \gamma_m) = p_1 + 6\,\text{ft}\,(62.4 - 52)\frac{\text{lb}}{\text{ft}^3} = p_1 + 62.4\frac{\text{lb}}{\text{ft}^2}$$

ENERGY 1-2 (no losses):
$$\frac{p_1}{\gamma} + \cancel{z_1} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \cancel{z_2} + \frac{V_2^2}{2g} \Rightarrow \frac{p_2 - p_1}{\gamma} = \frac{V_1^2 - V_2^2}{2g}$$

$$\frac{p_2 - p_1}{\gamma} = \frac{V_2^2}{2g}\left[\left(\frac{V_1}{V_2}\right)^2 - 1\right] = \frac{V_2^2}{2g}\left[\left(\frac{d_2}{d_1}\right)^4 - 1\right]$$

$$V_2 = \sqrt{\frac{2g(p_2 - p_1)/\gamma}{(d_2/d_1)^4 - 1}} = \sqrt{\frac{2 \times 32.2\,\text{ft/s}^2 \times 1.0\,\text{ft}}{(4/2)^4 - 1}} = 2.07\,\text{ft/s}$$

$$\frac{p_2 - p_1}{\gamma} = \frac{62.4\,\text{lb/ft}^2}{62.4\,\text{lb/ft}^3} = 1.0\,\text{ft}$$

continuity:
$$V_1 = V_2\left(\frac{d_2}{d_1}\right)^2 = 2.07\,\text{fps}\left(\frac{4}{2}\right)^2 = 8.28\,\text{fps}$$

$$V_3 = V_2\left(\frac{d_2}{d_3}\right)^2 = 2.07\,\text{fps}\left(\frac{4}{1}\right)^2 = 33.12\,\text{fps}$$

$$Q = V_1\frac{\pi d_1^2}{4} = 8.28\,\frac{\text{ft}}{\text{s}} \times \frac{\pi \times (2/12\,\text{ft})^2}{4} = 0.1806\,\text{cfs}$$

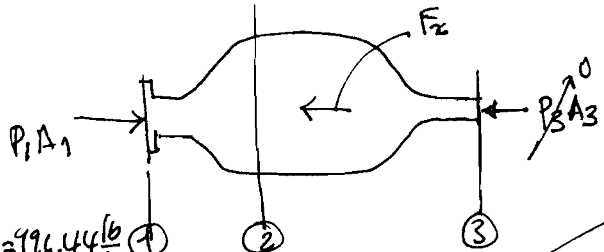
ENERGY 1-3 (no losses)
$$\frac{p_1}{\gamma} + \cancel{z_1} + \frac{V_1^2}{2g} = \cancel{\frac{p_3}{\gamma}}^{\,0} + \cancel{z_3} + \frac{V_3^2}{2g} \Rightarrow \frac{p_1}{\gamma} = \frac{V_3^2 - V_1^2}{2g}$$

$$p_1 = \frac{\gamma}{2g}(V_3^2 - V_1^2) = \frac{62.4\,\text{lb/ft}^3}{2 \times 32.2\,\text{ft/s}^2} \times (33.12^2 - 8.28^2)\frac{\text{ft}^2}{\text{s}^2} = 996.44\,\text{lb/ft}^2$$

also, $P_2 = P_1 + 62.4 \frac{lb}{ft^2} = (996.44 + 62.4)\frac{lb}{ft^2} = 1058.84 \frac{lb}{ft^2}$

MOMENTUM EQUATION

$\boxed{\text{Forces}}$ F_x : force from device on flow



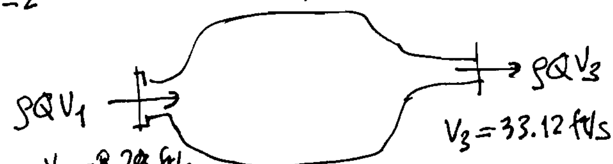
$$\Sigma F_x = P_1A_1 - F_x - 0$$
$$= P_1A_1 - F_x$$

$P_1 = 996.44 \frac{lb}{ft^2}$

$d_1 = 2\,in$

$\rightarrow Q = 0.1806\,cfs$

$\boxed{\text{Momentum flow}}$



$V_3 = 33.12\,ft/s$

$V_1 = 8.28\,ft/s$

$$\Delta(\rho Q V)_x = \text{out} - \text{in}$$
$$= \rho Q V_3 - \rho Q V_1$$
$$= \frac{\gamma}{g} Q (V_3 - V_1)$$

momentum equation: $\Sigma F_x = \Delta(\rho Q V)_x$:

$$P_1A_1 - F_x = \frac{\gamma}{g}Q(V_3 - V_1) \Rightarrow F_x = P_1A_1 - \frac{\gamma}{g}Q(V_3 - V_1)$$

$$F_x = 996.44\frac{lb}{ft^2} \times \frac{\pi \times (2/12\,ft)^2}{4} - \frac{62.4\,lb/ft^3}{32.2\,ft/s^2} \times 0.1806\frac{ft^3}{s} \times (33.12 - 8.28)\frac{ft}{s}$$

$\underline{F_x = 13.05\,lb \leftarrow}$ $\leftarrow$ FORCE FROM DEVICE ON FLOW

Force of flow on device: $F = 13.05\,lb \rightarrow$