

CEE 3500 – Fall 2005 – Assignment 11 - Solutions

5.26 In Fig. P5.26, assume water is flowing and neglect all head losses except at discharge. Find the flow rate if $h = 8$ ft. Assuming that $d = 10$ ft, the throat diameter is two-thirds the pipe diameter where it joins the downstream tank, and the atmospheric pressure is equal to the standard atmospheric pressure at 10,000 ft elevation, calculate the gage pressure and the absolute pressure in the constriction. The throat diameter is 14 in.

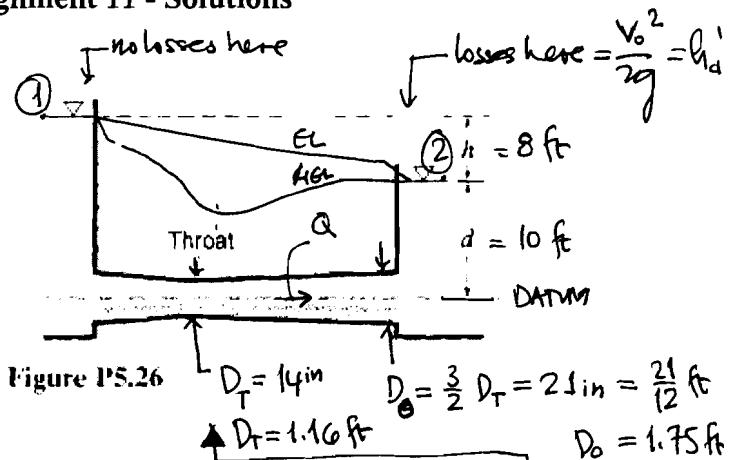


Figure P5.26 $D_T = 14$ in $D_0 = \frac{3}{2} D_T = 21$ in = $\frac{21}{12}$ ft $D_0 = 1.75$ ft

$h = 8$ ft $d = 10$ ft
at elevation 10,000 ft (Table A.3)
 $P_{atm} = 10.1083$ psia

~~The Throat is the location of the minimum pressure, which we make equal to the vapor pressure of water at, say, 60°F, i.e., $P_T = P_v = 0.256$ psia (NOT NEEDED)~~

$(P_1)_{gage} = 0$, $(P_1)_{abs} = P_{atm} = 10.1083$ psia

$V_1 = 0$, $z_1 = d + h = 18$ ft

$(P_2)_{gage} = 0$, $(P_2)_{abs} = P_{atm} = 10.1083$ psia

$V_2 = 0$, $z_2 = d = 10$ ft

ENERGY EQUATION ① → ②, no losses except for discharge losses

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_d' = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

using gage pressure

$$0 + 18 + 0 - \frac{V_0^2}{2g} = 0 + 10 + 0$$

$$\Rightarrow \frac{V_0^2}{2g} = 18 - 10 = 8 \Rightarrow V_0^2 = 2 \times 32.2 \times 8$$

$$\Rightarrow V_0^2 = 515.2 \Rightarrow V_0 = 22.69 \text{ fps}$$

$$Q = V_0 \cdot \frac{\pi D_0^2}{4} = 22.69 \times \frac{\pi \times 1.75^2}{4}$$

$Q = 54.6$ cfs

ENERGY EQN. ① - Throat, using gage pressure, $z_T = 0$

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_T}{\gamma} + z_T + \frac{V_T^2}{2g}$$

$$0 + 18 + 0 = \frac{P_T}{\gamma} + 0 + \frac{V_T^2}{2g}$$

use continuity $V_T = \frac{4Q}{\pi D_T^2} = \frac{4 \times 54.6}{\pi \times 1.16^2} = 51.66$ fps

$$\left(\frac{P_T}{\gamma}\right)_{gage} = 18 - \frac{V_T^2}{2g} = 18 - \frac{51.66^2}{2 \times 32.2} = -23.44 \text{ ft}$$

$$(P_T)_{gage} = -23.44 \times 62.4 = -1462.67 \frac{\text{lb}}{\text{ft}^2}$$

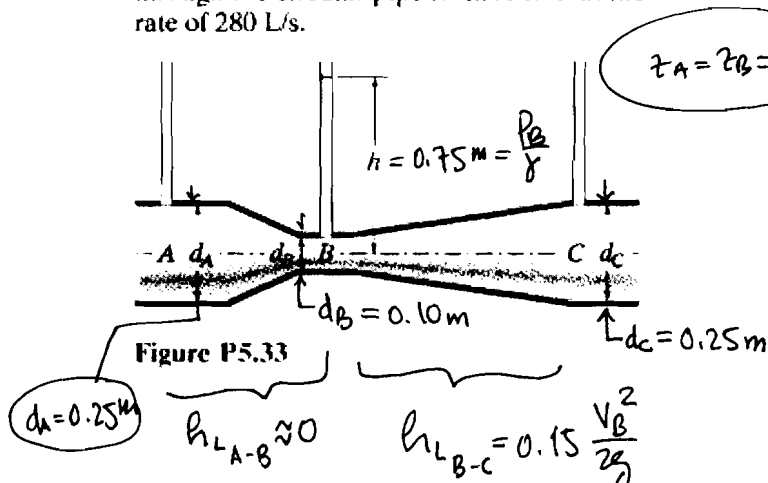
$$(P_T)_{gage} = -\frac{1462.67}{144} \text{ psi} = -10.15 \text{ psi}$$

$$(P_T)_{abs} = (P_T)_{gage} + P_{atm} = -10.15 + 10.10 =$$

$$(P_T)_{abs} = -0.05 \text{ (impossible)}$$

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5.33 In Fig. P5.33 friction loss between A and B is negligible while between B and C it is $0.15(V_B^2/2g)$. Given $h = 750$ mm, $d_A = d_C = 250$ mm, $d_B = 100$ mm. Find the pressure heads at A and C if the liquid is flowing through the circular pipe from A to C at the rate of 280 L/s.



$z_A = z_B = z_C$

(B-C) Energy equation

$$\frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g} - h_{L_{B-C}} = \frac{P_C}{\gamma} + z_C + \frac{V_C^2}{2g}$$

$$0.75 + \frac{35.65^2}{2 \times 9.81} - 0.15 \times \frac{35.65^2}{2 \times 9.81} = \frac{P_C}{\gamma} + \frac{5.70^2}{2 \times 9.81}$$

$$\frac{P_C}{\gamma} = 0.75 + \frac{35.65^2 - 0.15 \times 35.65^2 - 5.70^2}{2 \times 9.81}$$

$\frac{P_C}{\gamma} = 54.15\text{ m}$

$\frac{P_A}{\gamma} = ?$ $\frac{P_C}{\gamma} = ?$ $Q = 280\text{ L/s} = 0.28\text{ m}^3/\text{s}$

$$V_A = \frac{4Q}{\pi d_A^2} = \frac{4 \times 0.28}{\pi \times 0.25^2} = 5.70\text{ m/s}$$

$$V_B = \frac{4Q}{\pi d_B^2} = \frac{4 \times 0.28}{\pi \times 0.10^2} = 35.65\text{ m/s}$$

$$V_C = \frac{4Q}{\pi d_C^2} = \frac{4 \times 0.28}{\pi \times 0.25^2} = 5.70\text{ m/s}$$

(A-B) Bernoulli's theorem

$$\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g}$$

$$\frac{P_A}{\gamma} + \frac{5.70^2}{2 \times 9.81} = 0.75 + \frac{35.65^2}{2 \times 9.81}$$

$\frac{P_A}{\gamma} = 63.87\text{ m}$

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5.35 Referring to Fig. P5.34, assume the tube flows full and all friction losses are negligible. The diameter at B is 60 mm and the diameter of the jet discharging into the air is 80 mm. If $h = 5$ m, what is the flow rate? What is the pressure head at B? What would be the flow rate if the tube were cut off at B?

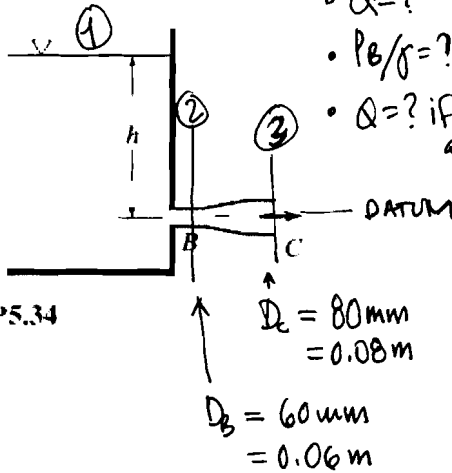


Figure P5.34

- $Q = ?$
- $P_B/\gamma = ?$
- $Q = ?$ if cutoff at B

$z_1 = h = 5$ m	$z_2 = 0$	$z_3 = 0$
$P_1 = 0$	$P_2 = ?$	$P_3 = 0$
$V_1 = 0$	$V_2 = \frac{4Q}{\pi D_B^2}$	$V_3 = \frac{4Q}{\pi D_C^2}$
	\Downarrow	\Downarrow
	$\frac{V_2^2}{2g} = \frac{8Q^2}{g\pi^2 D_B^4}$	$\frac{V_3^2}{2g} = \frac{8Q^2}{g\pi^2 D_C^2}$

Bernoulli theorem ①-③

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

$$0 + h + 0 = 0 + 0 + \frac{8Q^2}{g\pi^2 D_C^2}$$

$$Q^2 = \frac{\pi^2 g D_C^4 h}{8} = \frac{\pi^2 \times 9.81 \times 0.08^4 \times 5}{8} = 2.48 \times 10^{-3}$$

$$Q = 4.97 \times 10^{-2} \text{ m}^3/\text{s} = 49.79 \text{ L/s}$$

Bernoulli theorem ①-②

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$0 + h + 0 = \frac{P_2}{\gamma} + 0 + \frac{8Q^2}{\pi^2 g D_B^2}$$

$$\frac{P_B}{\gamma} = \frac{P_2}{\gamma} = h - \frac{8Q^2}{\pi^2 g D_B^2} = 5 - \frac{8 \times 4.97 \times 10^{-2}}{\pi^2 \times 9.81 \times 0.06^2}$$

$$\frac{P_B}{\gamma} = 3.86 \text{ ft}$$

If nozzle is cut off at B, then

$$P_B = 0, z_B = 0, V_B = \frac{4Q}{\pi D_B^2} \Rightarrow \frac{V_B^2}{2g} = \frac{8Q^2}{\pi^2 D_B^2}$$

Bernoulli theorem ①-②

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$0 + h + 0 = 0 + 0 + \frac{8Q^2}{\pi^2 D_B^2}$$

$$Q^2 = \frac{\pi^2 g h D_B^2}{8} = \frac{\pi^2 \times 9.81 \times 5 \times 0.06^2}{8}$$

$$Q^2 = 7.842 \times 10^{-4}$$

$$Q = 2.8 \times 10^{-2} \text{ m}^3/\text{s} = 28.00 \text{ L/s}$$

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5.36 In Fig. P5.36 friction losses in the pipe below pump P are $1.8V^2/2g$ with the barometer pressure at 12.50 psia. The liquid in the suction pipe has a velocity of 7 fps. What would be the maximum allowable value of z if the liquid were (a) water at 70°F; (b) gasoline at a vapor pressure of 9 psia with a specific weight of 47 lb/ft³?

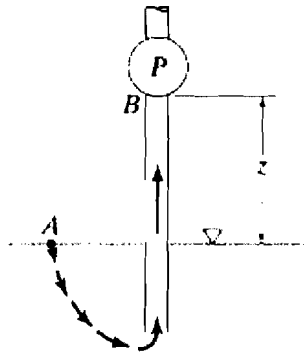


Figure P5.36

$$\Rightarrow z = \frac{P_{\text{atm}} - P_v}{\gamma} - 2.8 \frac{V^2}{2g}$$

↓ in psia

$$z = \frac{(12.50 - P_v) \times 144}{\gamma} - 2.13$$

(a) water at 70°F $\Rightarrow P_v = 0.363$ psia $\gamma = 62.30$ lb/ft³

$$z = \frac{(12.50 - 0.363) \times 144}{62.30} - 2.13 = \frac{1710.8}{62.30} - 2.13 = 25.92 \text{ ft}$$

(b) gasoline, $P_v = 9$ psia, $\gamma = 47$ lb/ft³

$$z = \frac{(12.50 - 9) \times 144}{47} - 2.13 = \frac{504}{47} - 2.13 = 8.59 \text{ ft}$$

$$P_{\text{atm}} = 12.50 \text{ psia}, V = 7 \text{ fps}$$

~~at point A~~

$$\text{at point A} \Rightarrow (P_A)_{\text{gage}} = 0, z_A = 0, V_A = 0$$

$$\text{at point B} \Rightarrow (P_B)_{\text{gage}} = P_v + P_{\text{atm}}, z_B = z, V_B = V$$

$$h_{L_{AB}} = 1.8 \frac{V^2}{2g}$$

Energy equation A \rightarrow B

$$\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} - h_{L_{A-B}} = \frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g}$$

$$0 + 0 + 0 - 1.8 \frac{V^2}{2g} = \frac{P_v - P_{\text{atm}}}{\gamma} + z + \frac{V^2}{2g}$$

$$P_{\text{atm}} = 12.50 \text{ psia} = 12.50 \times 144 \frac{\text{lb}}{\text{ft}^2} = 1800 \frac{\text{lb}}{\text{ft}^2}$$

$$\frac{V^2}{2g} = \frac{7^2}{2 \times 32.2} = 0.76 \text{ ft}, 2.8 \frac{V^2}{2g} = 2.13$$

◀ NOTE: z is referred to as the NPSH (Net Positive Suction Head) for the pump.