Lecture 22

Numerical Solution for Manifold Location

I. Introduction

- In the previous lecture it was seen how the optimal manifold location can be determined semi-graphically using a set of non-dimensional curves for the uphill and downhill laterals
- This location can also be determined numerically
- In the following, equations are developed to solve for the unknown length of the uphill lateral, x_u, without resorting to a graphical solution

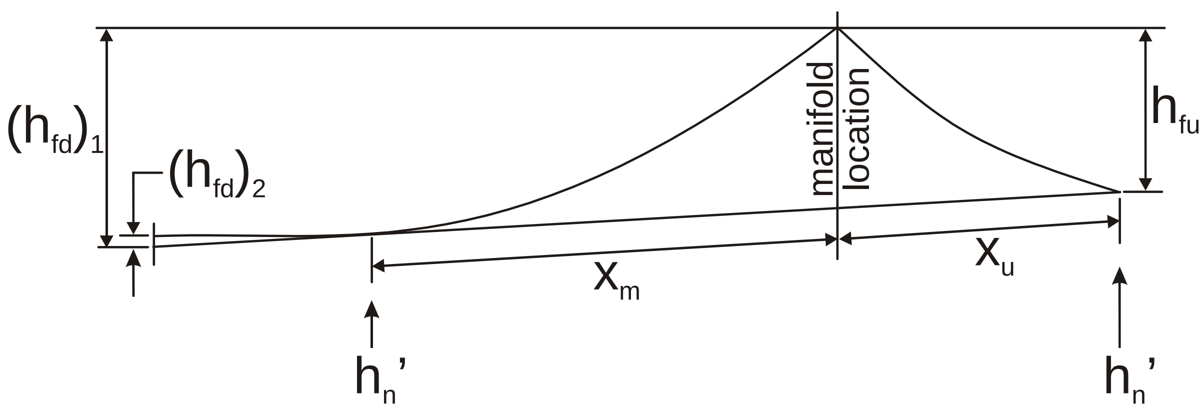
II. Definition of Minimum Lateral Head

- In the uphill lateral, the minimum head is at the closed end of the lateral (furthest uphill location in the subunit)
- This minimum head is equal to:

$$h_n' = h_l - h_{fu} - x_u S \tag{394}$$

where h_n' is the minimum head (m); h_l is the lateral inlet head (m); h_{fu} is the total friction loss in the uphill lateral (m); x_u is the length of the uphill lateral (m); and S is the slope of the ground surface (m/m)

- Note that S must be a positive value



- In the downhill lateral, the minimum head may be anywhere from the inlet to the outlet, depending on the lateral hydraulics and the ground slope
- The minimum head in the downhill lateral is equal to:

$$h_n' = h_l - (h_{fd})_1 + (h_{fd})_2 + x_m S \tag{395}$$

where $(h_{fu})_1$ is the total friction loss in the downhill lateral (m); $(h_{fu})_2$ is the friction loss from the closed end of the downhill lateral to the location of minimum head (m); and x_m is the distance from the manifold (lateral inlet) to the location of minimum head in the downhill lateral (m)

- Combining Eqs. 1 and 2:

$$h_{fu} + S\left(x_u + x_m\right) - \left(h_{fd}\right)_1 + \left(h_{fd}\right)_2 = 0 \tag{396}$$

III. Location of Minimum Head in Downhill Lateral

- The location of minimum head is where the slope of the ground surface, S, equals the friction loss gradient, J':

$$S = J' \tag{397}$$

where both S and J' are in m/m, and S is positive (you can take the absolute value of S)

- Using the Hazen-Williams equation, the friction loss gradient in the downhill lateral (at the location where S = J') is:

$$J' = \left[\frac{S_e + f_e}{S_e}\right]\left[1.212(10)^{10}\left(\frac{q_a\left(L - x_u - x_m\right)}{3{,}600\,S_e\,C}\right)^{1.852} D^{-4.87}\right] \tag{398}$$

where: J' is the friction loss gradient (m/m);
S_e is the emitter spacing on the laterals (m);
f_e is the equivalent lateral length for emitter head loss (m);
q_a is the nominal emitter discharge (lph);
L is the sum of the lengths of the uphill and downhill laterals (m);
x_u is the length of the uphill lateral (m);
x_m is the distance from the manifold to the location of minimum head in
the downhill lateral (m);
C is approximately equal to 150 for plastic pipe; and
D is the lateral inside diameter (mm);

- The value of 3,600 is to convert q_a units from lph to lps
- Note that $x_d = L - x_u$, where x_d is the length of the downhill lateral
- Note that $q_a(L - x_u - x_m)/(3{,}600\ S_e)$ is the flow rate in the lateral, in lps, at the location of minimum head, x_m meters downhill from the manifold
- Combining the above two equations, and solving for x_m:

$$x_m = L - x_u - \left(\frac{0.0129\,S_e C D^{2.63}}{q_a}\right)\left(\frac{S_e S}{S_e + f_e}\right)^{0.54} \qquad (399)$$

where the permissible values of x_m are: $0 \le x_m \le x_d$

- Combine Eqs. 396 & 399, and solve for x_u by iteration
- Alternatively, based on Eq. 8.7a from the textbook, x_m can be defined as:

$$x_m = L - x_u - \left[\frac{3{,}600\,S_e}{q_a}\right]\left[\left(\frac{S D^{4.75}}{7.89(10)^5}\right)\left(\frac{S_e}{S_e + f_e}\right)\right]^{0.571} \qquad (400)$$

IV. Definition of Head Loss Gradients

- In the uphill lateral, the head loss is:

$$h_{fu} = J_u' F_u x_u \qquad (401)$$

- In the downhill lateral, the head losses are:

$$\left(h_{fd}\right)_1 = J_{d1}' F_{d1}\left(L - x_u\right) \qquad (402)$$

and,

$$\left(h_{fd}\right)_2 = J_{d2}' F_{d2}\left(L - x_u - x_m\right) \qquad (403)$$

- The above three “F” values are as defined by Eq. 8.9 in the textbook
- The friction loss gradients (in m/m) are:

$$J_u' = K_J\left(x_u\right)^{1.852} \qquad (404)$$

$$J_{d1}' = K_J\left(L - x_u\right)^{1.852} \qquad (405)$$

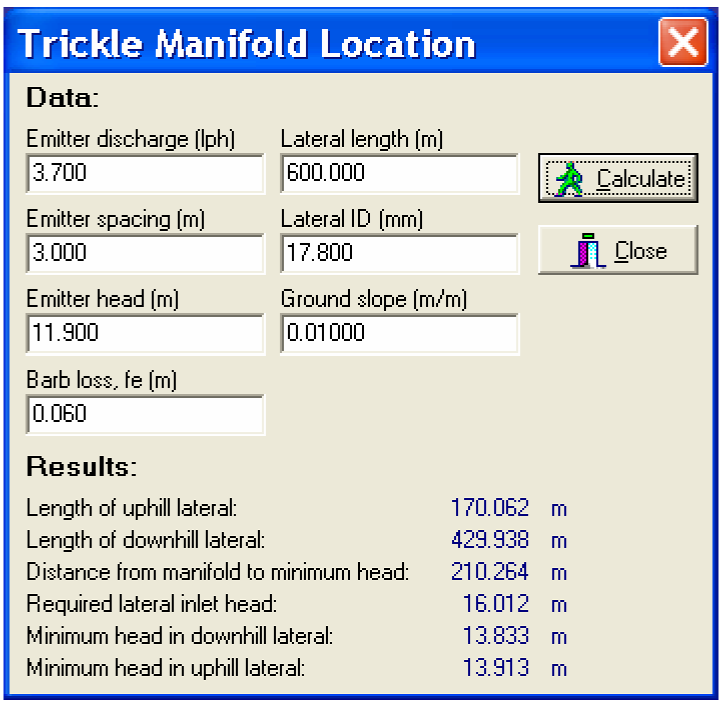
$$J_{d2}' = K_J\left(L - x_u - x_m\right)^{1.852} \qquad (406)$$

where, for the Hazen-Williams equation,

$$K_J = \left(\frac{S_e + f_e}{S_e}\right)\left[1.212(10)^{10} D^{-4.87}\left(\frac{q_a}{3{,}600\, S_e C}\right)^{1.852}\right] \qquad (407)$$

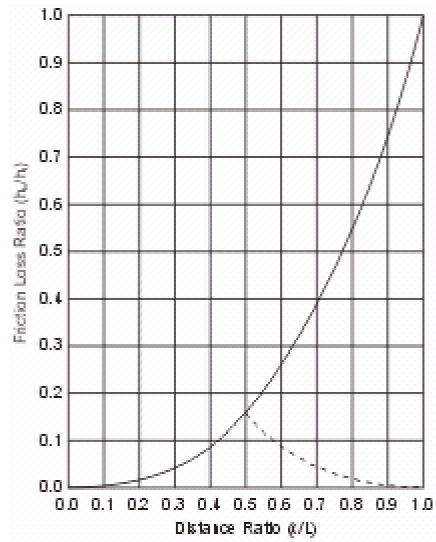
V. Solving for Optimal Manifold Location

- Using the definitions above, solve for the length of the uphill lateral, x_u
- Then, $x_d = L - x_u$
- Note that you might prefer to use the Darcy-Weisbach and Blasius equations for the manifold calculations; they may be more accurate than Hazen-Williams
- The “OptManifold” computer program uses the Darcy-Weisbach & Blasius equations



Where do these Equations Come From?

I. Derivation of Nondimensional Friction Loss Curves



- The nondimensional friction loss curves are actually one curve, with the lower half laterally inverted and shown as a dashed line (Fig. 8.2)
- The dashed line is simply for flow in the opposite direction, which for our purposes is in the uphill direction
- We know from the previous lectures and from intuition that the uphill segment of lateral pipe will not be more than ½ the total length, because it is equal to ½ for the case where the ground slope is zero

- Following is the derivation for Eq. 8.10b, from which Fig. 8.2 was plotted

- Darcy-Weisbach equation for circular pipes:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{408}$$

- Blasius equation, for estimating f for small diameter (D < 125 mm) "smooth pipes" (e.g. PE & PVC), and based on more complete equations that are used to plot the Moody diagram

$$f \cong 0.32 N_R^{-0.25} \tag{409}$$

where N_R is the Reynolds number, which for circular pipes is:

$$N_R = \frac{VD}{\nu} = \frac{4Q}{\nu \pi D} \tag{410}$$

- The kinematic viscosity, ν, is equal to about $1.003(10)^{-6}$ m^2/s for water at 20°C
- Then, for this kinematic viscosity,

$$f \cong 0.32 \left(\frac{4Q}{\nu \pi D} \right)^{-0.25} \approx 0.0095 \left(\frac{Q}{D} \right)^{-0.25} \tag{411}$$

- Putting the above into the Darcy-Weisbach equation:

$$h_f = 0.0095\left(\frac{Q}{D}\right)^{-0.25}\frac{L}{D}\frac{V^2}{2g} \tag{412}$$

or:

$$h_f \cong 0.00079L\frac{Q^{1.75}}{D^{4.75}} \tag{413}$$

where h_f is in m; L is in m; Q is in m^3/s; and D is in m

- Eq. 8.7a is obtained by having Q in lps, and D in mm, whereby the above coefficient changes to $7.9(10)^7$

- Finally, in the above, use L(x/L) instead of L, and Q(x/L) instead of Q, and call it "h_{fx}":

$$h_{fx} \cong 0.00079L(x/L)\frac{\left[Q(x/L)\right]^{1.75}}{D^{4.75}} \tag{414}$$

- Then,

$$\frac{h_{fx}}{h_f} = (x/L)(x/L)^{1.75} = (x/L)^{2.75} \tag{415}$$

which is Eq. 8.10b and the basis for the nondimensional friction loss curves, valid for plastic pipes with D < 125 mm

II. Derivation of Equation for ΔH_c

- The difference between the minimum pressure head and the pressure head at the closed end of a lateral, ΔH_c, is used to calculate the minimum head in the lateral, H_n'
- This is because the pressure head at the end of the lateral is easily calculated as:

$$H_c = H_l - h_f - \Delta h_e \tag{416}$$

where Δh_e is negative for downhill slopes

- But the minimum pressure head does not necessarily occur at the end of the lateral when the lateral runs downhill
- Thus, in general,

$$H'_n = H_c - \Delta H_c \tag{417}$$

- The above is from Eq. 22.7 in the textbook
- These concepts can also be interpreted graphically as in Fig. 22.1
- Following is a derivation of an equation for ΔHc (based on Keller and Rodrigo 1979)

1. The minimum pressure in the lateral occurs where the ground slope (for a uniform slope) equals the slope of the friction loss curve. The dimensionless friction loss curve is defined as (Eq. 8.10b or Eq. 22.3b):

$$\left(\frac{h_{fx}}{h_f}\right)_{pair} = \left(\frac{x}{L}\right)^{2.75} \tag{418}$$

2. The slope of this friction loss curve is:

$$\frac{d\left(\frac{h_{fx}}{h_f}\right)_{pair}}{d\left(\frac{x}{L}\right)} = 2.75\left(\frac{x}{L}\right)^{1.75} \tag{419}$$

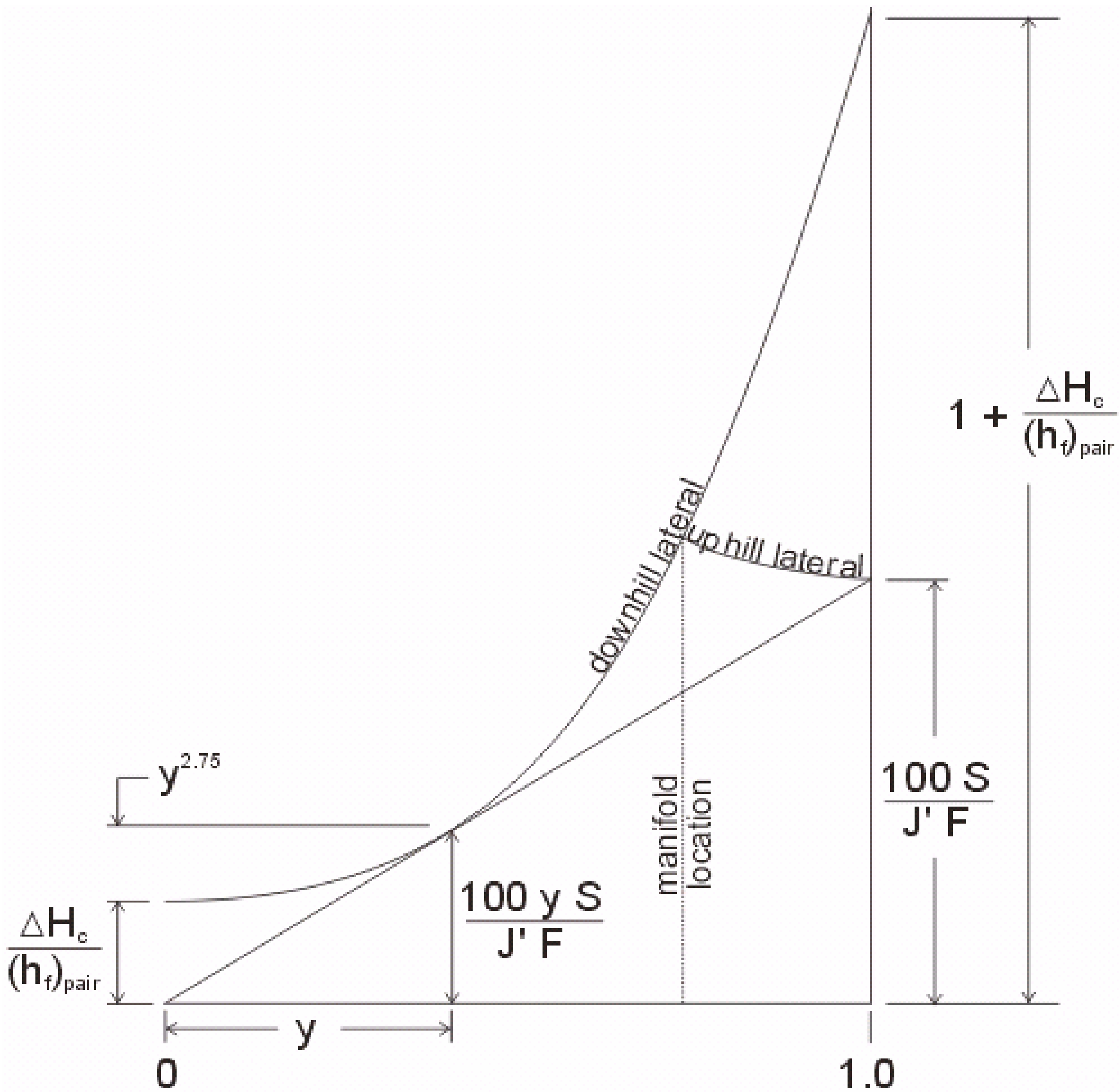
3. The uniform ground slope on the dimensionless graph is:

$$\left(\frac{\Delta h_e}{h_f}\right)_{pair} = \frac{SL}{\left(\frac{J'FL}{100}\right)} = \frac{100\,S}{J'F} \tag{420}$$

4. Then,

$$\frac{100\,S}{J'F} = 2.75\left(y\right)^{1.75} \tag{421}$$

in which y is the value of x/L where the minimum pressure occurs ($0 \le y \le 1$); S is the ground slope (m/m); J’ is the friction loss gradient for the flow rate in the pair of laterals (m/100 m); and F is the reduction coefficient for multiple outlet pipes (usually about 0.36)



5. Solve for y:

$$y = \left(\frac{100\,S}{2.75\,J'F} \right)^{1/1.75} \tag{422}$$

or,

$$y \approx \left(\frac{100\,S}{J'} \right)^{1/1.75} \tag{423}$$

where F ≈ 0.36

6. Referring to the figure on the previous page, the following equality can be written:

$$y^{2.75} + \frac{\Delta H_c}{(h_f)_{pair}} = \frac{100\,y\,S}{J'F} \tag{424}$$

solving for ΔH_c,

$$\Delta H_c = (h_f)_{pair}\left(\frac{100\,y\,S}{J'F} - y^{2.75}\right) \tag{425}$$

where,

$$(h_f)_{pair} = \frac{J'FL}{100} \tag{426}$$

and y can be approximated as in step 5 above (for F = 0.36)

7. After manipulating the equation a bit, the following expression is obtained:

$$\Delta H_c = 8.9\,L\,S^{1.57}\,(J')^{-0.57} \tag{427}$$

for ΔH_c in m; L in m; S in m/m; and J' in m/100 m. Note that J' and L are for the <u>pair</u> of laterals, not only uphill or only downhill

III. Derivation of Equation for α

- The parameter α is used in the calculation of inlet pressure for a pair of laterals on sloping ground where (Eq. 22.17):

$$H_l = H_a + \alpha (h_f)_{pair} + \left(\frac{x}{L} - 0.5\right)(\Delta h_e)_{pair} \tag{428}$$

with,

$$H_a = \left(\frac{q_a}{K_d}\right)^{1/x} \tag{429}$$

$$(h_f)_{pair} = \frac{J'FL}{100} \tag{430}$$

$$\left(\Delta h_e\right)_{pair} = \frac{100\,S}{J'F}\left(h_f\right)_{pair} = SL \tag{431}$$

- Note that $(\Delta h_e)_{pair}$ must be a negative number
- The ratio x/L is the distance to the manifold, where L is the length of the <u>pair of laterals</u>
- The following derivation is based on equations presented by Keller and Rodrigo (1979):

  1. Given that for a single lateral approximately ¾ of the friction loss occurs from the inlet to the point where the average pressure occurs (multiple outlets, uniform outlet spacing, constant discharge from outlets, single lateral pipe size) we have the following:

$$\alpha\left(h_f\right)_{pair} = \frac{3}{4}\left(h_f\right)_{downhill}\left(\frac{x}{L}\right) + \frac{3}{4}\left(h_f\right)_{uphill}\left(1-\frac{x}{L}\right) \tag{432}$$

The above equation is *a weighted average* because the uphill lateral is shorter than the downhill lateral

2. Recall that,

$$\left(\frac{h_{fx}}{h_f}\right)_{pair} = \left(\frac{x}{L}\right)^{2.75} \tag{433}$$

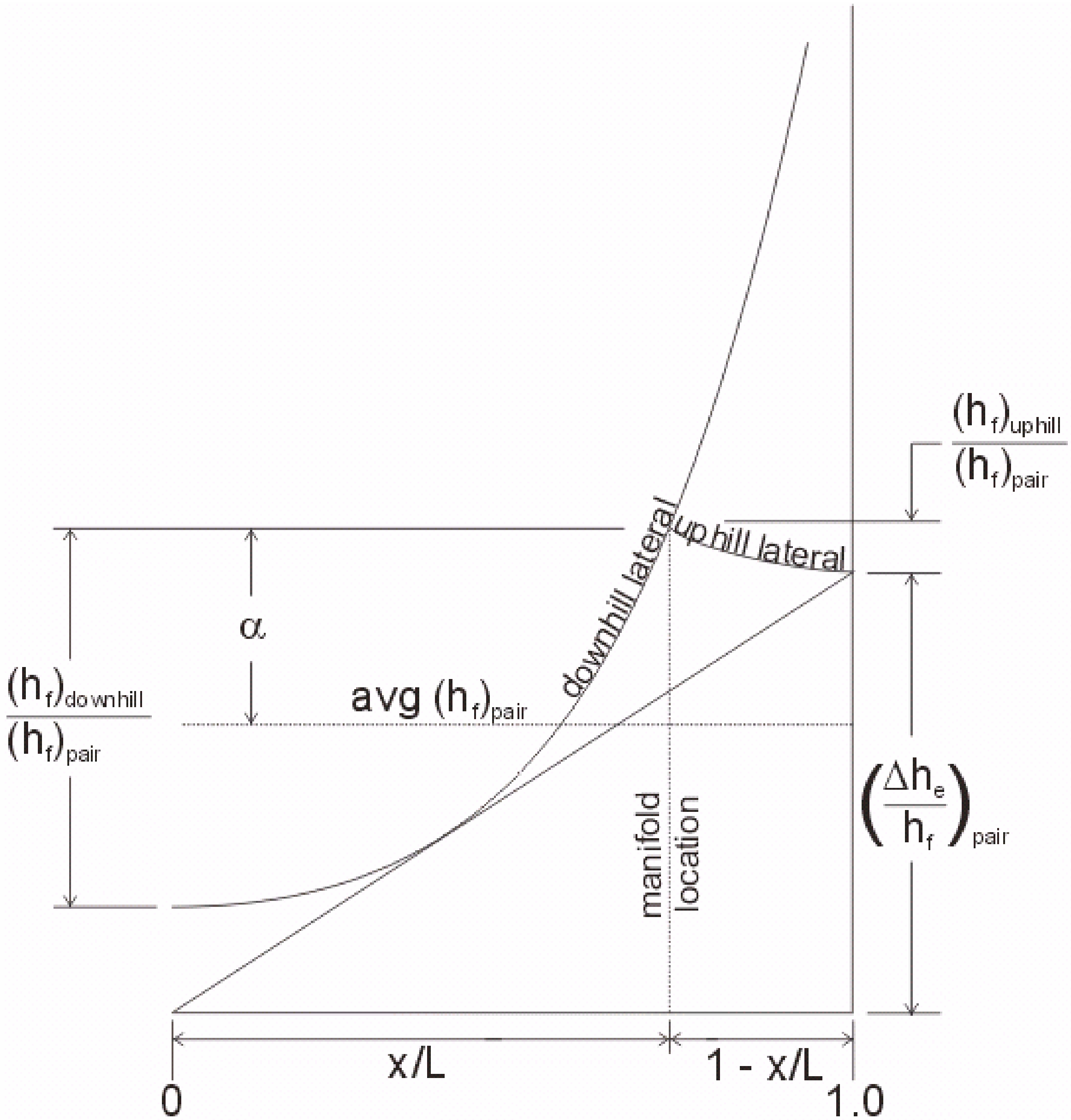
Then,

$$\begin{aligned}\left(h_f\right)_{downhill} &= \left(\frac{x}{L}\right)^{2.75}\left(h_f\right)_{pair}\\ \left(h_f\right)_{uphill} &= \left(1-\frac{x}{L}\right)^{2.75}\left(h_f\right)_{pair}\end{aligned} \tag{434}$$

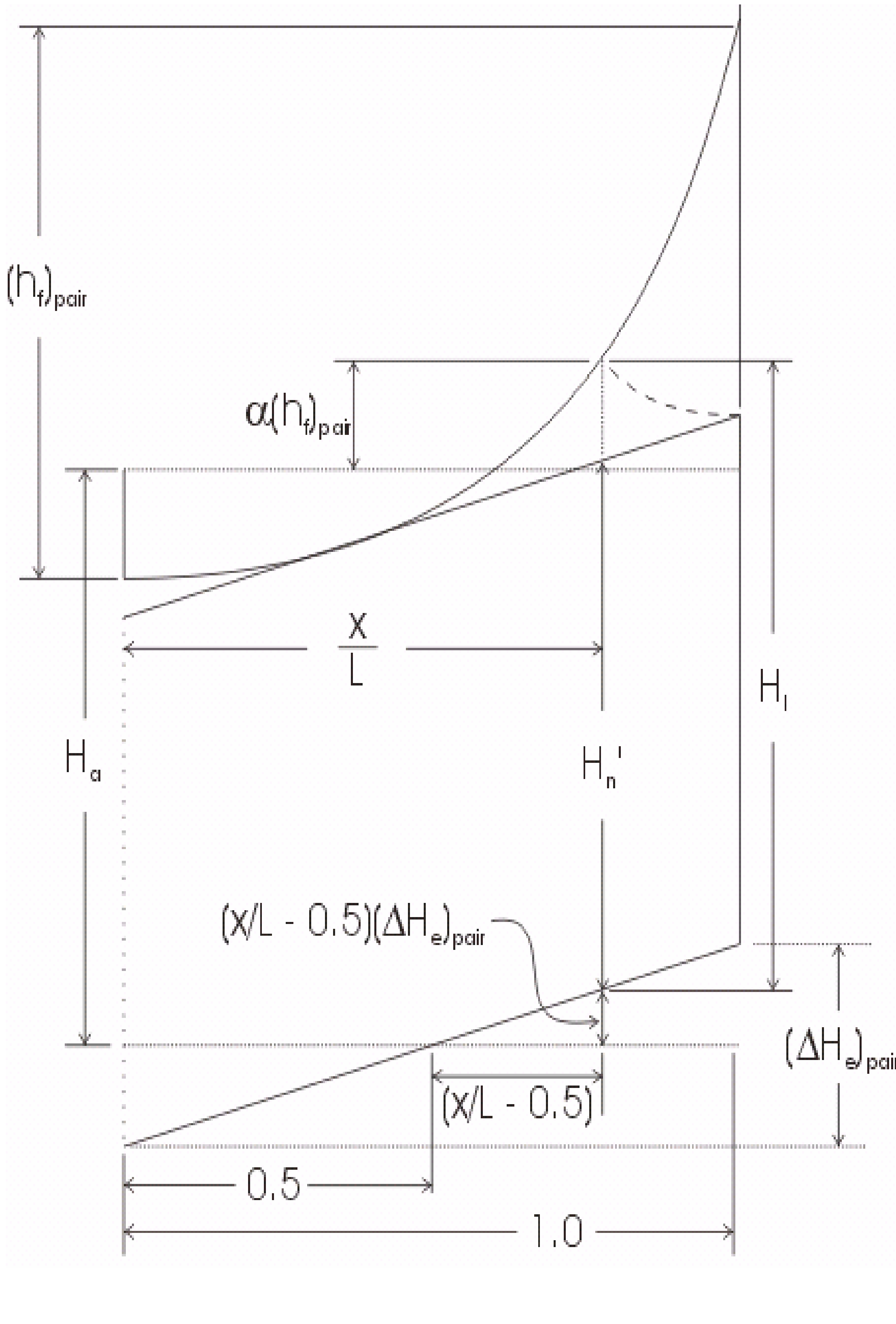
3. Combining equations:

$$\alpha\left(h_f\right)_{pair} = \frac{3}{4}\left(h_f\right)_{pair}\left[\left(\frac{x}{L}\right)\left(\frac{x}{L}\right)^{2.75} + \left(1-\frac{x}{L}\right)\left(1-\frac{x}{L}\right)^{2.75}\right] \tag{435}$$

$$\alpha = \frac{3}{4}\left[\left(\frac{x}{L}\right)^{3.75} + \left(1-\frac{x}{L}\right)^{3.75}\right] \tag{436}$$



- This last equation for α is Eq. 22.25 from the textbook
- See the figure below


(h_f)_pair
α(h_f)_pair
X
L
H_a
H_n'
H_l
(X/L - 0.5)(ΔH_e)_pair
(ΔH_e)_pair
(X/L - 0.5)
0.5
1.0