Lecture 11

Pumps & System Curves

I. Pump Efficiency and Power

- Pump efficiency, E_{pump}

$$E_{pump} = \frac{\text{water horsepower}}{\text{brake horsepower}} = \frac{WHP}{BHP} \tag{221}$$

where *brake horsepower* refers to the input power needed at the pump shaft (not necessarily in “horsepower”; could be watts or some other unit)

- Pump efficiency is usually given by the pump manufacturer
- Typically use the above equation to calculate required BHP, knowing E_{pump}
- Water horsepower is defined as:

$$WHP = \frac{QH}{3956} \tag{222}$$

where WHP is in horsepower; Q in gpm; and H in feet of head. The denominator is derived from:

$$\gamma QH = \frac{\left(62.4\text{ lbs/ft}^3\right)\left(\text{gal/min}\right)\left(\text{ft}\right)}{\left(33{,}000\text{ ft-lbs/min-HP}\right)\left(7.481\text{ gal/ft}^3\right)} \approx \frac{QH}{3956} \tag{223}$$

where $\gamma = \rho g$, and ρ is water density. In metric units:

$$WHP = \rho g QH = \frac{\left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left(\text{l/s}\right)\left(\text{m}\right)}{\left(1000\text{ l/m}^3\right)\left(1000\text{ W/kW}\right)} = \frac{QH}{102} \tag{224}$$

where WHP is in kW; Q in lps; and H in meters of head

$$1\text{ HP} = 0.746\text{ kW} \tag{225}$$

- *Total Dynamic Head*, TDH, is defined as:

$$TDH = \Delta Elev + h_f + \frac{P}{\gamma} + \frac{V^2}{2g} \tag{226}$$

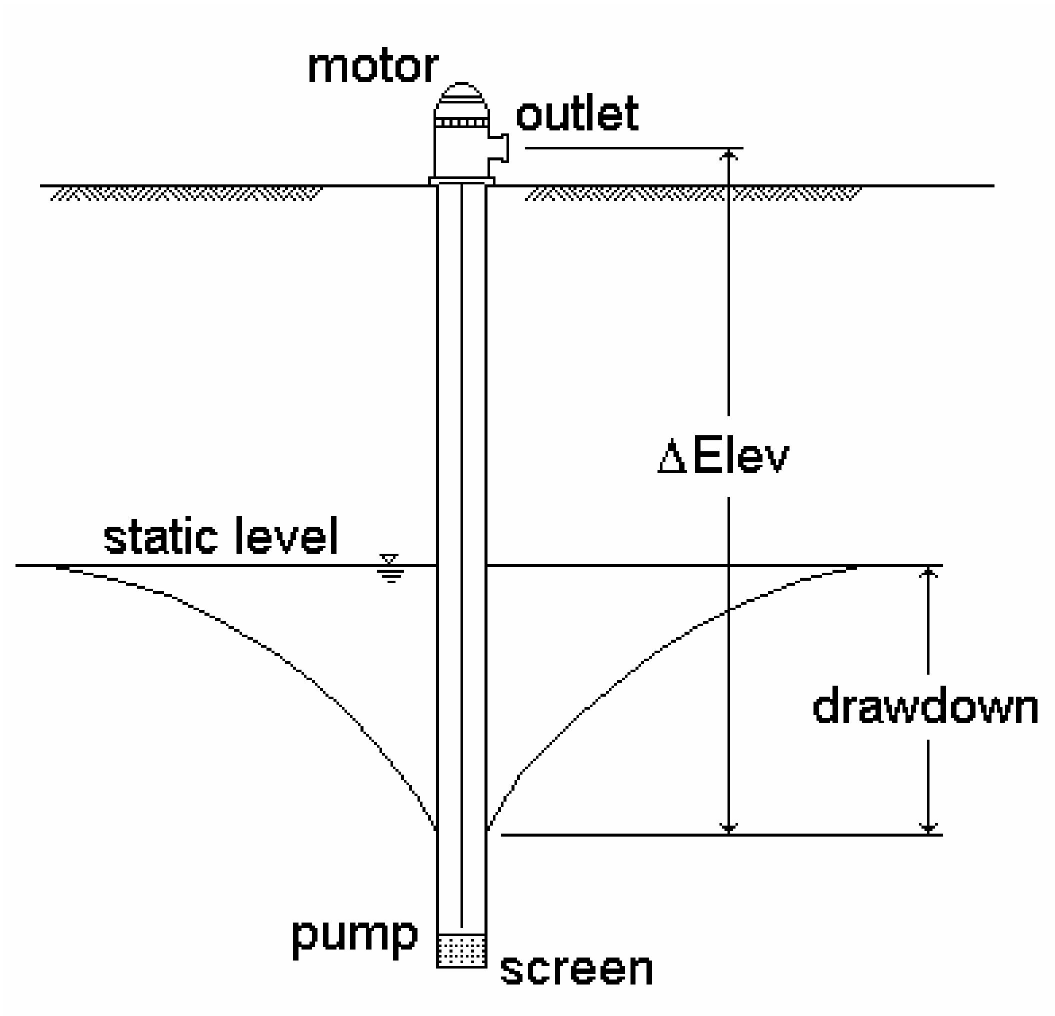
where the pressure, P, and velocity, V, are measured at the pump outlet, and h_f is the total friction loss from the entrance to the exit, including minor losses

- At zero flow, with the pump running,

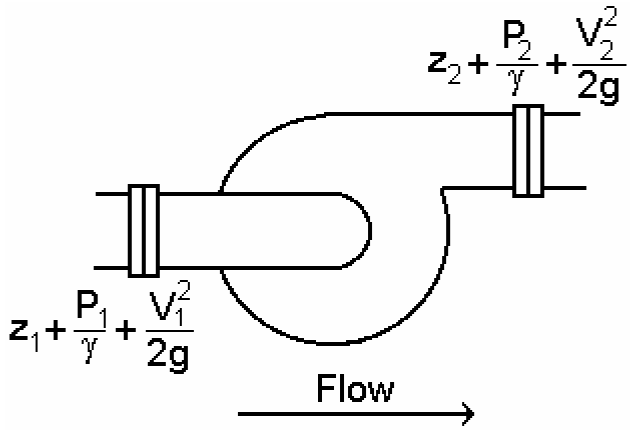
$$TDH = \Delta Elev + \frac{P}{\gamma} \tag{227}$$

but recognizing that in some cases P/γ is zero for a zero flow rate

- The elevation change, ΔElev, is positive for an increase in elevation (i.e. lifting the water)
- Consider a turbine pump in a well:

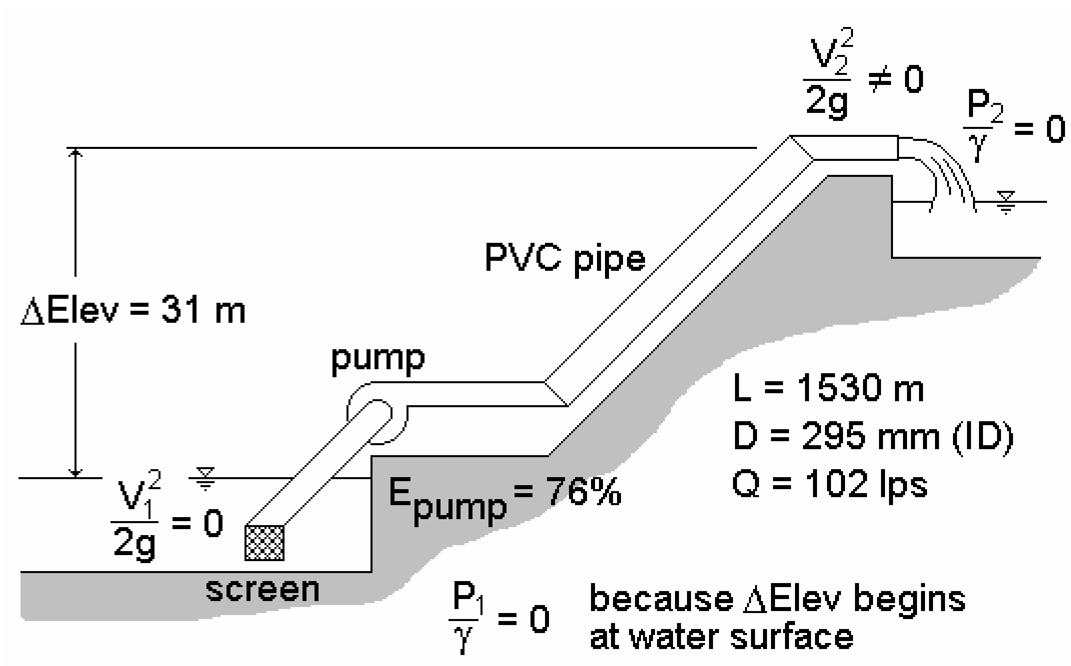


Consider a centrifugal pump:



II. Example TDH & WHP Calculation

- Determine TDH and WHP for a centrifugal pump discharging into the air...



Head loss due to friction:

$$h_f = h_{screen} + 3h_{elbow} + h_{pipe} \tag{228}$$

for PVC, $\varepsilon \approx 1.5(10)^{-6}$ m, relative roughness is:

$$\frac{\varepsilon}{D} = \frac{1.5(10)^{-6}}{0.295} = 0.0000051 \tag{229}$$

Average velocity,

$$V = \frac{Q}{A} = \frac{4(0.102)}{\pi(0.295)^2} = 1.49 \text{ m/s} \tag{230}$$

Reynolds number, for 10°C water:

$$N_R = \frac{VD}{\nu} = \frac{(1.49 \text{ m/s})(0.295 \text{ m})}{1.306(10)^{-6} \text{ m}^2/\text{s}} = 336{,}600 \tag{231}$$

- From the Moody diagram, f = 0.0141
- From the Blasius equation, f = 0.0133
- From the Swamee-Jain equation, f = 0.0141 (same as Moody)

Using the value from Swamee-Jain,

$$h_{pipe} = f\frac{L}{D}\frac{V^2}{2g} = 0.0141\left(\frac{1{,}530}{0.295}\right)\frac{(1.49)^2}{2(9.81)} = 8.27 \text{ m} \tag{232}$$

Water Temperature (°C)	**Kinematic Viscosity (m^2/s)**
0	0.000001785
5	0.000001519
10	0.000001306
15	0.000001139
20	0.000001003
25	0.000000893
30	0.000000800
40	0.000000658
50	0.000000553
60	0.000000474

The values in the above table can be closely approximated by:

$$\nu = \left(83.9192\,T^2 + 20{,}707.5\,T + 551{,}173\right)^{-1} \tag{233}$$

where T is in ºC; and ν is in m^2/s

From Table 11.2, for a 295-mm (12-inch) pipe and long radius 45-deg flanged elbow, the K_r value is 0.15

$$h_{elbow} = K_r \frac{V^2}{2g} = (0.15)\frac{(1.49)^2}{2(9.81)} = (0.15)(0.11) = 0.017 \text{ m} \qquad (234)$$

For the screen, assume a 0.2 m loss. Then, the total head loss is:

$$h_f = 0.2 + 3(0.017) + 8.27 = 8.5 \text{ m} \qquad (235)$$

With the velocity head of 0.11 m, the total dynamic head is:

$$TDH = 31 + 8.5 + 0.11 \approx 40 \text{ m} \qquad (236)$$

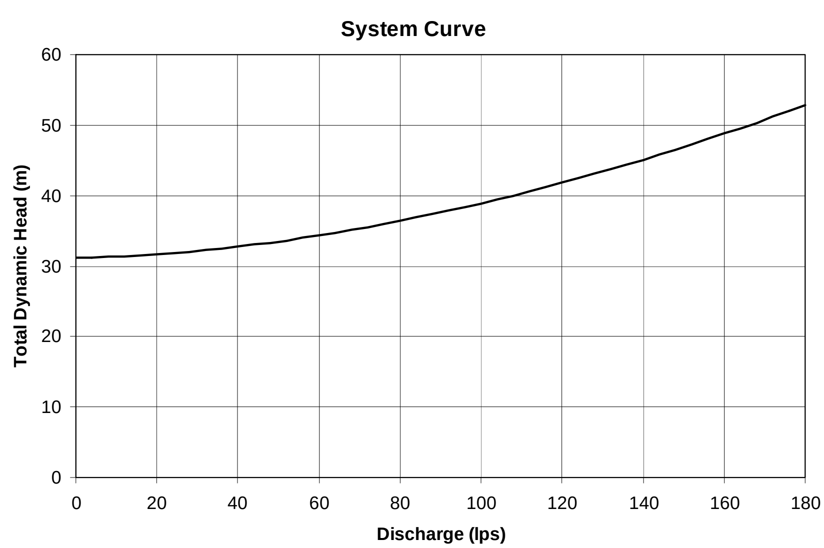
The water horsepower is:

$$WHP = \frac{QH}{102} = \frac{(102 \text{ lps})(40 \text{ m})}{102} = 40 \text{ kW (54 HP)} \qquad (237)$$

The required brake horsepower is:

$$BHP = \frac{WHP}{E_{pump}} = \frac{40 \text{ kW}}{0.76} \approx 53 \text{ kW (71 HP)} \qquad (238)$$

- This BHP value would be used to select a motor for this application
- These calculations give us one point on the system curve (Q and TDH)
- In this simple case, there would be only one system curve:

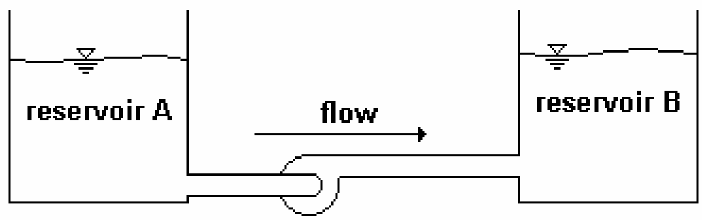


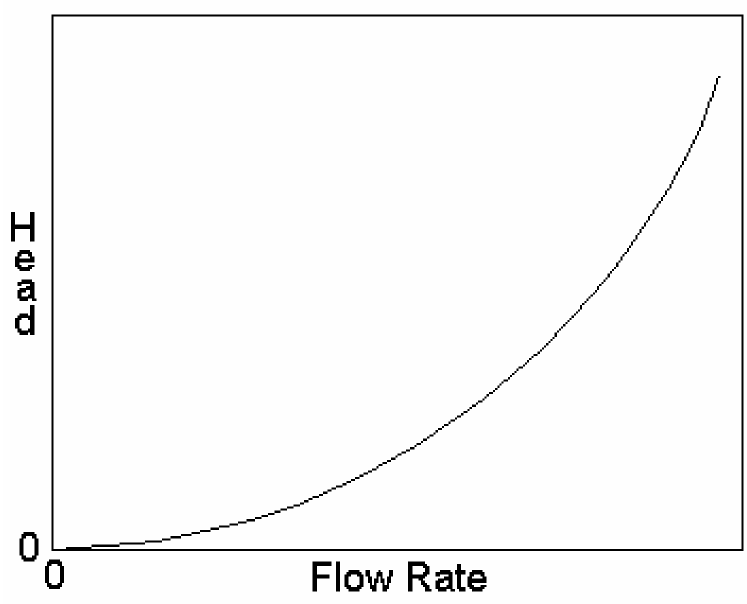
III. System Curves

- The “system curve” is a graphical representation of the relationship between discharge and head loss in a system of pipes
- The system curve is completely independent of the pump characteristics
- The basic shape of the system curve is parabolic because the exponent on the head loss equation (and on the velocity head term) is 2.0, or nearly 2.0
- The system curve will start at zero flow and zero head if there is no static lift, otherwise the curve will be vertically offset from the zero head value

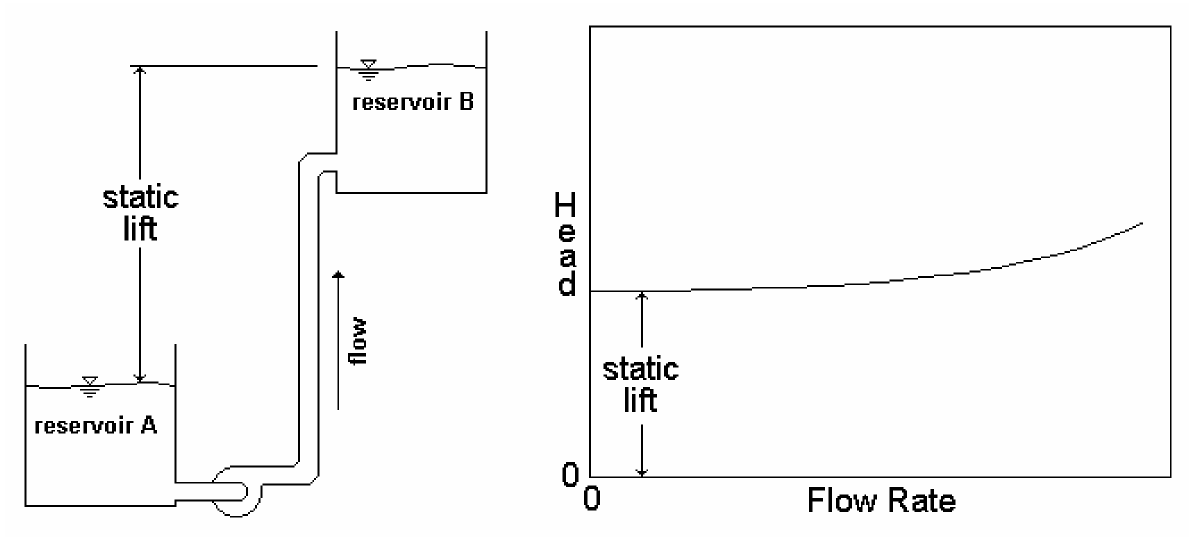
- Most sprinkle and trickle irrigation systems have more than one system curve because either the sprinklers move between sets (periodic-move systems), move continuously, or “stations” (blocks) of laterals are cycled on and off
- The intersection between the system and pump characteristic curves is the operating point (Q and TDH)
- A few examples of system curves:

1. All Friction Loss and No Static Lift

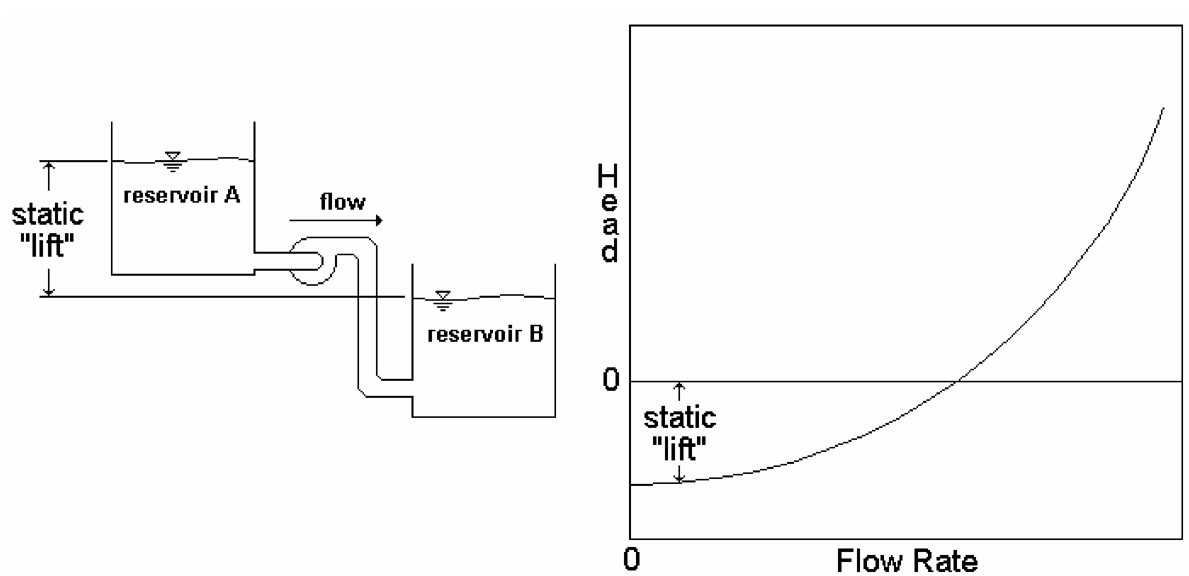




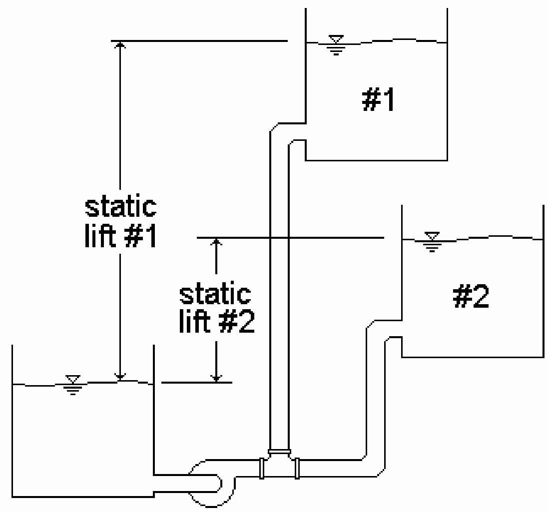
2. Mostly Static Lift, Little Friction Loss

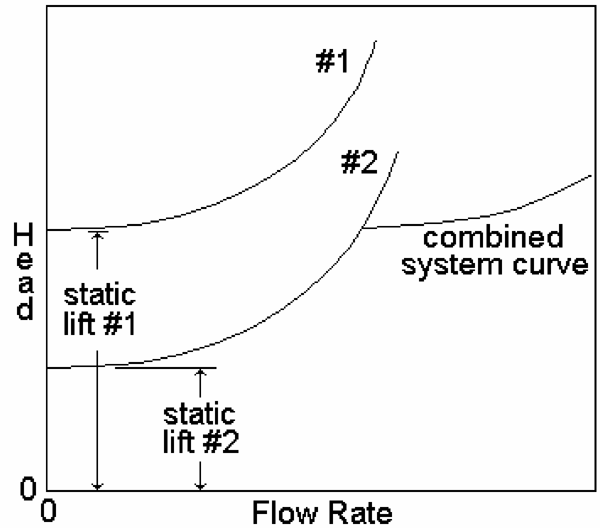


3. Negative Static Lift



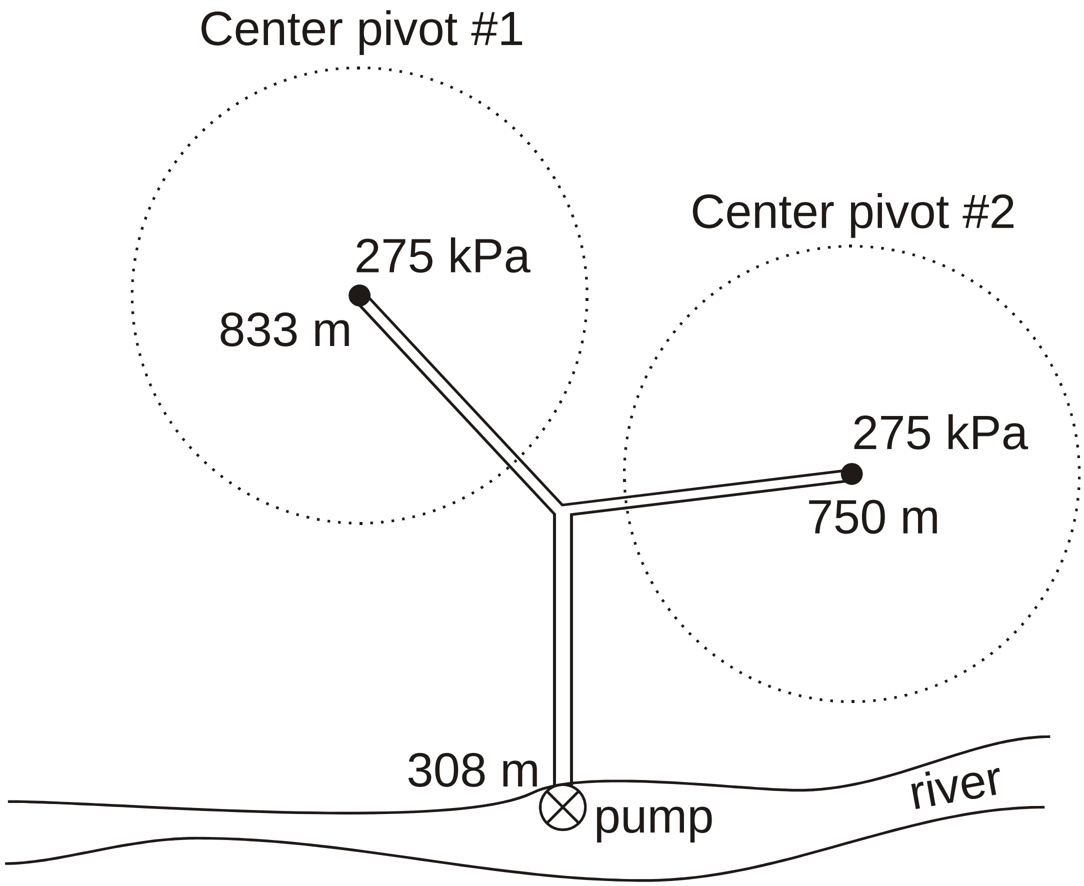
4. Two Different Static Lifts in a Branching Pipe





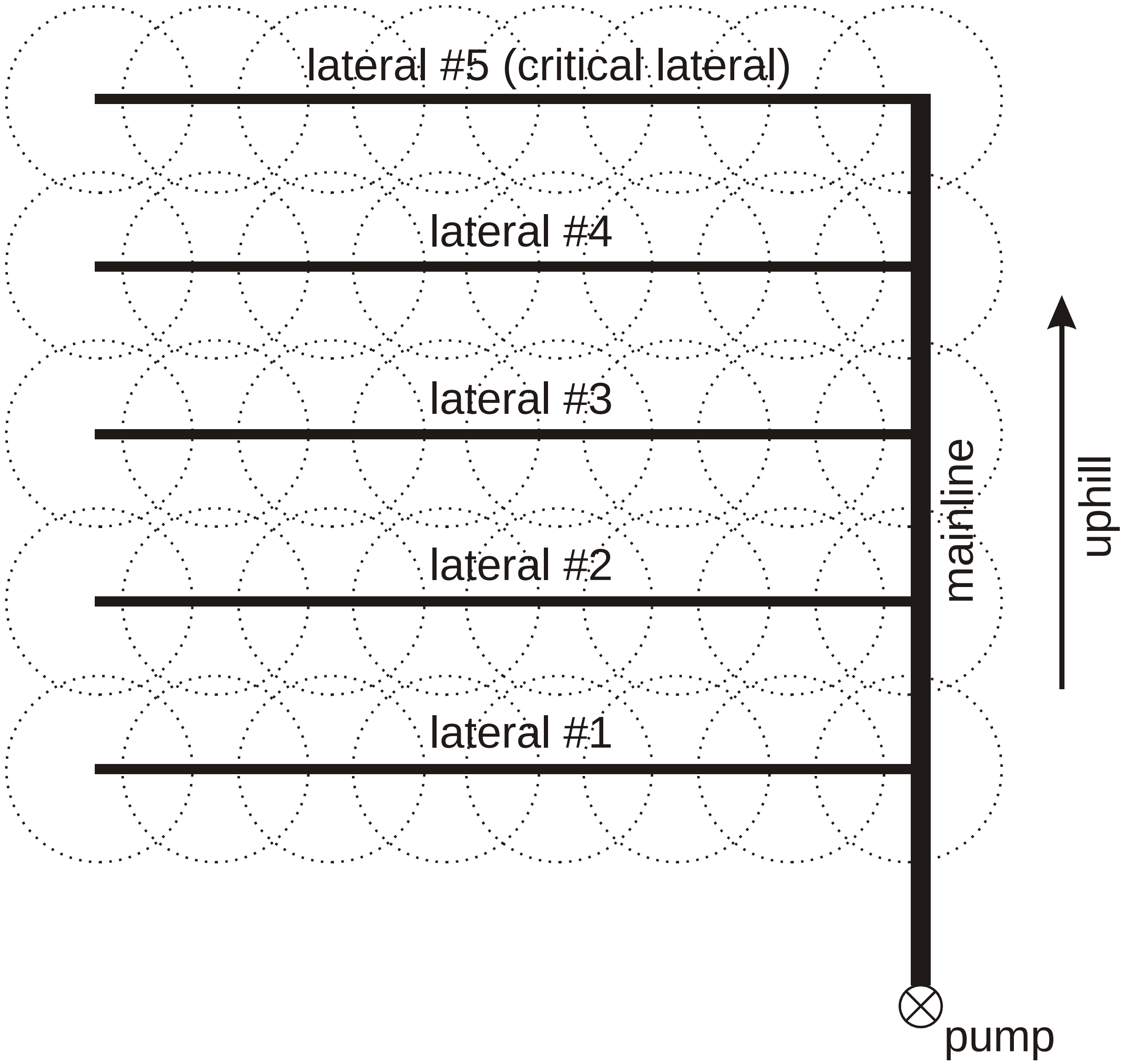
5. Two Center Pivots in a Branching Pipe Layout

- The figure below shows two center pivots supplied by a single pump on a river bank
- One of the pivots (#1) is at a higher elevation than the other, and is further from the pump – it is the "critical" branch of the two-branch pipe system
- Center pivot #2 will have excess pressure when the pressure is correct at Center pivot #1, meaning it will need pressure regulation at the inlet to the pivot lateral
- Use the critical branch (the path to Center pivot #1, in this case) when calculating TDH for a given operating condition – <u>Do</u> <u>Not</u> <u>Follow</u> <u>Both</u> <u>Branches</u> when calculating TDH
- if you cannot determine which is the critical branch by simple inspection, you must test different branches by making calculations to determine which is the critical one
- Note that the system curve will change with center pivot lateral position when the topography is sloping and or uneven within the circle
- Of course, the system curve will also be different if only one of the center pivots is operating



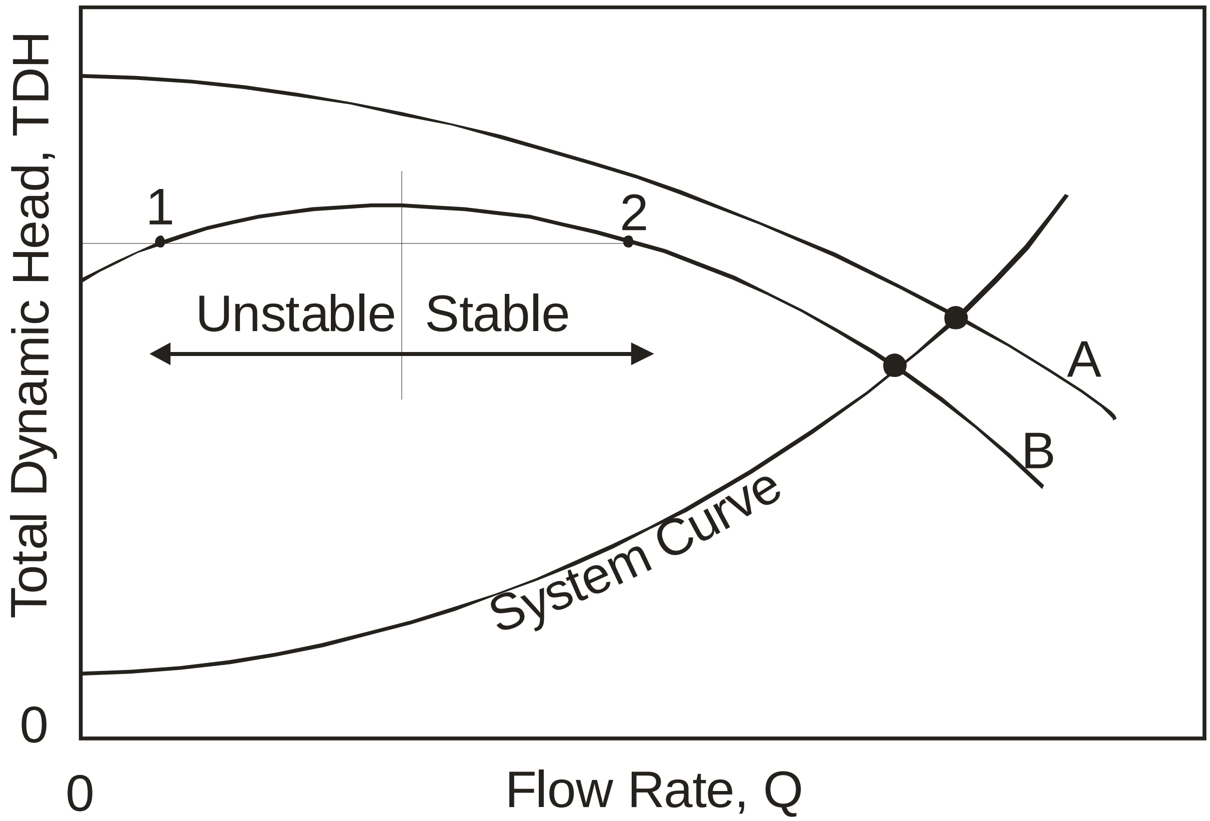
6. A Fixed Sprinkler System with Multiple Operating Laterals

- The next figure shows a group of laterals in parallel, attached to a common mainline in a fixed sprinkler system
- All of the sprinklers operate at the same time (perhaps for frost control or crop cooling purposes, among other possibilities)
- This is another example of a branching pipe system
- Since the mainline runs uphill, it is easy to determine by inspection that the furthest lateral will be the critical branch in this system layout – use this branch to determine the TDH for a given system flow rate
- Hydraulic calculations would be iterative because you must also determine the flow rate to each of the laterals since the flow rate is changing with distance along the mainline
- But in any case, <u>Do</u> <u>Not</u> <u>Follow</u> <u>Multiple</u> <u>Branches</u> when determining the TDH for a given system flow rate
- Remember that TDH is the resistance "felt" by the pump for a given flow rate and system configuration



7. Two Flow Rates for Same Head on Pump Curve

- Consider the following graph
- “A” has a unique Q for each TDH value
- “B” has two flow rates for a given head, over a range of TDH values
- Pumps with a characteristic curve like “B” should usually be avoided



Affinity Laws and Cavitation

I. Affinity Laws

1. Pump operating speed:

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \qquad \frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \qquad \frac{BHP_1}{BHP_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{239}$$

where Q is flow rate; N is pump speed (rpm); H is head; and BHP is "brake horsepower"

- The first relationship involving Q is valid for most pumps
- The second and third relationships are valid for centrifugal, mixed-flow, and axial-flow pumps

2. Impeller diameter:

$$\frac{Q_1}{Q_2} = \frac{D_1}{D_2} \qquad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad \frac{BHP_1}{BHP_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{240}$$

- These three relationships are valid only for centrifugal pumps
- These relationships are not as accurate as those involving pump operating speed, N (rpm)

Comments:

- The affinity laws are only valid within a certain range of speeds, impeller diameters, flow rates, and heads
- The affinity laws are more accurate near the region of maximum pump efficiency (which is where the pump should operate if it is selected correctly)

- It is more common to apply these laws to reduce the operating speed or to reduce the impeller diameter (diameter is never increased)
- We typically use these affinity laws to fix the operating point by shifting the pump characteristic curve so that it intersects the system curve at the desired Q and TDH

II. Fixing the Operating Point

Combine the first two affinity law relationships to obtain:

$$\frac{H_1}{H_2} = \left(\frac{Q_1}{Q_2}\right)^2 \tag{241}$$

- If this relationship is plotted with the pump characteristic curve and the system curve, it is called the "equal efficiency curve"
- This is because there is typically only a small change in efficiency with a small change in pump speed
- Note that the "equal efficiency curve" will pass through the origin (when Q is zero, H is zero)

- Follow these steps to adjust the: (1) speed; or, (2) impeller diameter, such that the actual operating point shifts up or down along the system curve:

    1. Determine the head, H_2, and discharge, Q_2, at which the system should operate (the desired operating point)
    2. Solve the above equation for H_1, and make a table of H_1 versus Q_1 values (for fixed H_2 and Q_2):

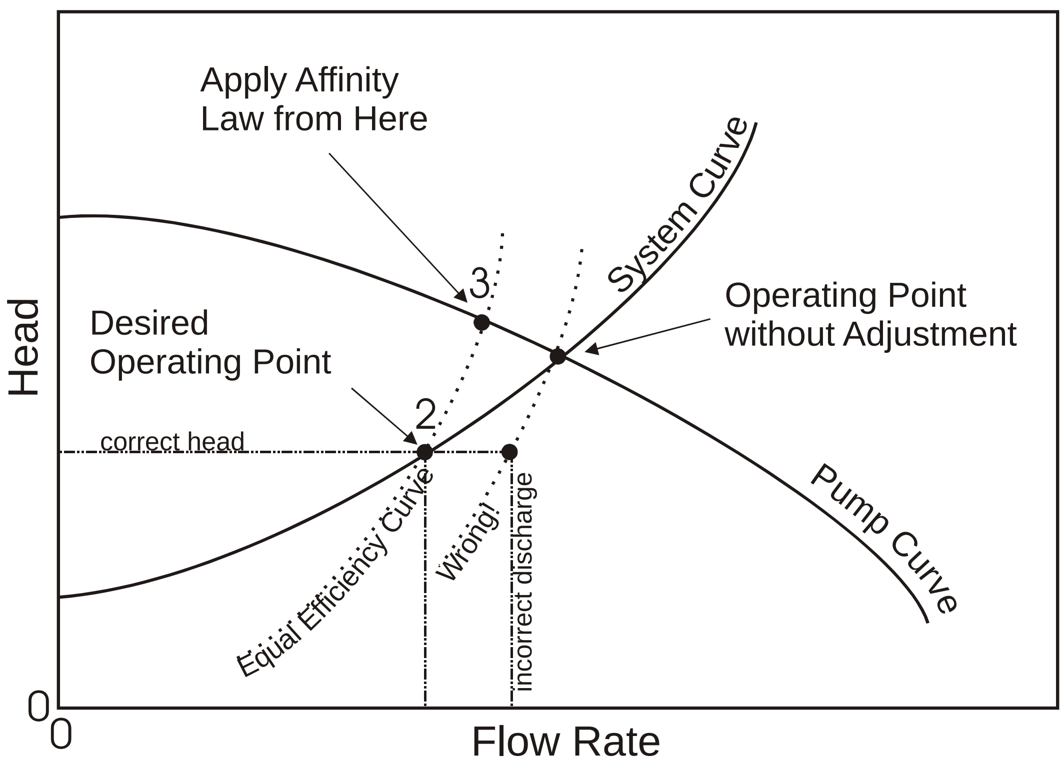
$$H_1 = H_2 \left(\frac{Q_1}{Q_2} \right)^2 \tag{242}$$

3. Plot the values from this table on the graph that already has the pump characteristic curve
4. Locate the intersection between the pump characteristic curve and the "equal efficiency curve", and determine the Q_3 and H_3 values at this intersection
5. Use either of the following equations to determine the new pump speed (or use equations involving D to determine the trim on the impeller):

$$N_{new} = N_{old} \left(\frac{Q_2}{Q_3} \right) \quad \text{or,} \quad N_{new} = N_{old} \sqrt{\frac{H_2}{H_3}} \tag{243}$$

6. Now your actual operating point will be the desired operating point (at least until the pump wears appreciably or other physical changes occur)

- You cannot directly apply any of the affinity laws in this case because you will either get the right discharge and wrong head, or the right head and wrong discharge



III. Specific Speed

- The specific speed is a dimensionless index used to classify pumps
- It is also used in pump design calculations

Pump Type	***Specific Speed***
Centrifugal (volute case)	500 - 5,000
Mixed Flow	4,000 - 10,000
Axial Flow	10,000 - 15,000

- To be truly dimensionless, it is written as:

$$N_s = \frac{2\pi N\sqrt{Q}}{(gH)^{0.75}} \tag{244}$$

where the 2π is to convert revolutions (dimensional) to radians (dimensionless)

- Example: units could be N = rev/s; Q = m^3/s; g = m/s^2; and H = m
- However, in practice, units are often mixed, the 2π is not included, and even g may be omitted
- This means that N_s must not only be given numerically, but the exact definition must be specified

IV. Cavitation

- Air bubbles will form (the water boils) when the pressure in a pump or pipeline drops below the vapor pressure
- If the pressure increases to above the vapor pressure downstream, the bubbles will collapse
- This phenomenon is called “cavitation”
- Cavitation often occurs in pumps, hydroelectric turbines, pipe valves, and ship propellers
- Cavitation is a problem because of the energy released when the bubbles collapse; formation and subsequent collapse can take place in only a few thousandths of a second, causing local pressures in excess of 150,000 psi, and local speeds of over 1,000 kph
- The collapse of the bubbles has also been experimentally shown to emit small flashes of light (“sonoluminescence”) upon implosion, followed by rapid expansion on shock waves
- Potential problems:

  1. noise and vibration
  2. reduced efficiency in pumps

3. reduced flow rate and head in pumps
4. physical damage to impellers, volute case, piping, valves

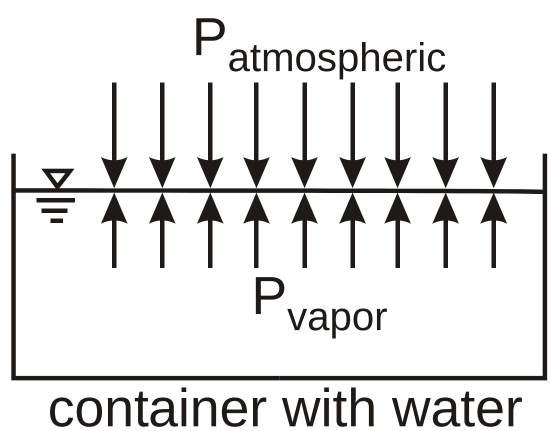
- From a hydraulics perspective cavitation is to be avoided
- But, in some cases cavitation is desirable. For example,

1. acceleration of chemical reactions
2. mixing of chemicals and or liquids
3. ultrasonic cleaning

- Water can reach the boiling point by:

1. reduction in pressure (often due to an increase in velocity)
2. increase in temperature

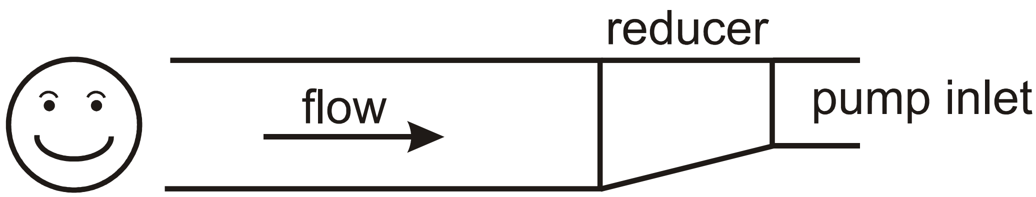
- At sea level, water begins to boil at 100°C (212°F)
- But it can boil at lower temperatures if the pressure is less than that at mean sea level (14.7 psi, or 10.34 m)

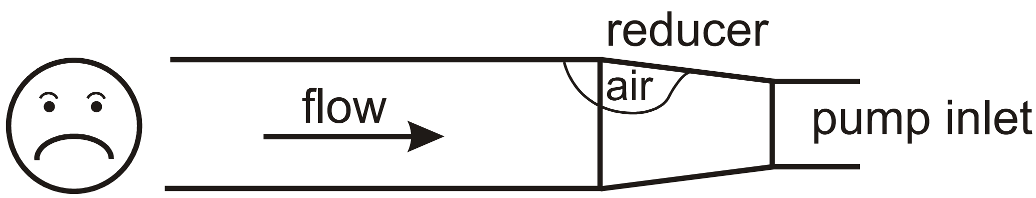


- Pump inlets often have an eccentric reducer (to go from a larger pipe diameter to the diameter required at the pump inlet:

1. Large suction pipe to reduce friction loss and increase NPSHa, especially where $NPSH_a$ is already too close to $NPSH_r$ (e.g. high-elevation pump installations where the atmospheric pressure head is relatively low)
2. Eccentric reducer to avoid accumulation of air bubbles at the top of the pipe

- See the following figure…





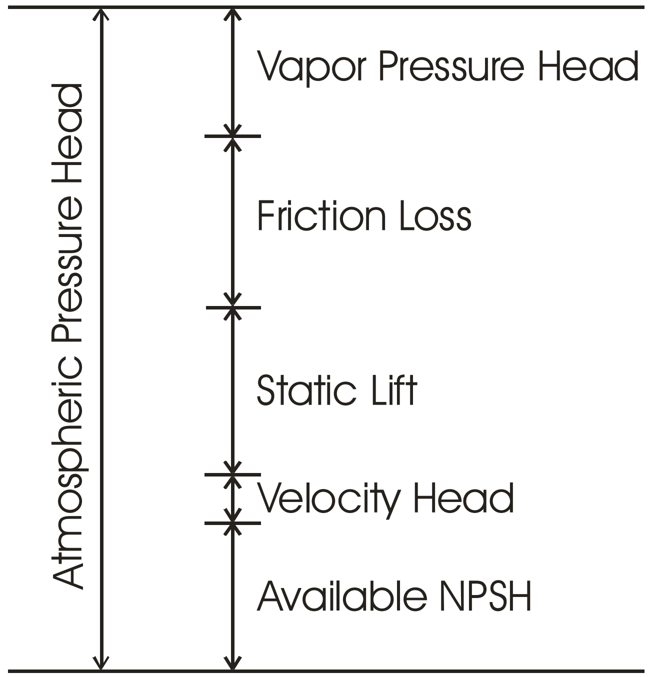
Required NPSH

- Data from the manufacturer are available for most centrifugal pumps
- Usually included in this data are recommendations for required *Net Positive Suction Head*, $NPSH_r$
- $NPSH_r$ is the minimum pressure head at the entrance to the pump, such that cavitation does not occur in the pump
- The value depends on the type of pump, its design, and size
- $NPSH_r$ also varies with the flow rate at which the pump operates
- $NPSH_r$ generally increases with increasing flow rate in a given pump
- This is because higher velocities occur within the pump, leading to lower pressures
- Recall that according to the Bernoulli equation, pressure will tend to decrease as the velocity increases, elevation being the same
- $NPSH_r$ is usually higher for larger pumps, meaning that cavitation can be more of a problem in larger pump sizes

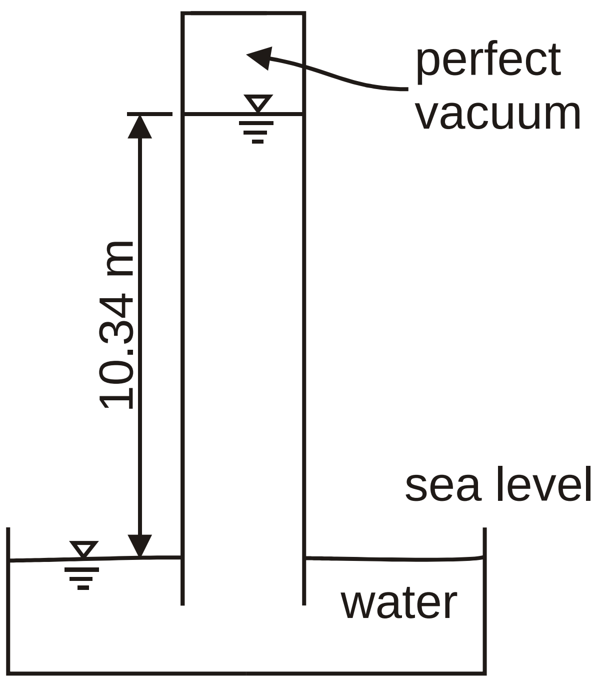
Available NPSH

- The available NPSH, or $NPSH_a$, is equal to the atmospheric pressure minus all losses in the suction piping (upstream side of the pump), vapor pressure, velocity head in the suction pipe, and static lift
- When there is suction at the pump inlet (pump is operating, but not yet primed), the only force available to raise the water is that of the atmospheric pressure
- But, the suction is not perfect (pressure does not reduce to absolute zero in the pump) and there are some losses in the suction piping

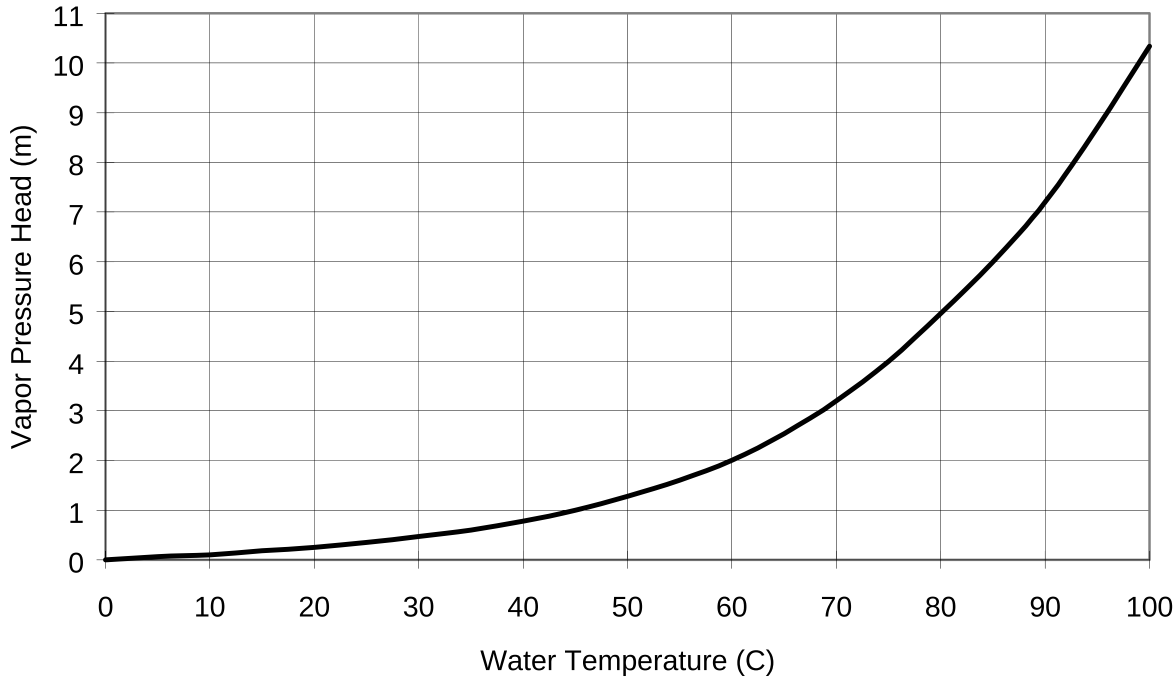
$$NPSH_a = h_{atm} - h_{vapor} - h_f - h_{lift} - \frac{V^2}{2g} \qquad (245)$$



- If the pump could create a "perfect vacuum" and there were no losses, the water could be "sucked up" to a height of 10.34 m (at mean sea level)
- Average atmospheric pressure is a function of elevation above msl



- 10.34 m is equal to 14.7 psi, or 34 ft of head
- Vapor pressure of water varies with temperature



- Herein, when we say "vapor pressure," we mean "saturation vapor pressure"
- Saturation vapor pressure head (as in the above graph) can be calculated as follows:

$$h_{vapor} = 0.0623 \exp\left(\frac{17.27\,T}{T + 237.3}\right) \tag{246}$$

for h_{vapor} in m; and T in ºC

- Mean atmospheric pressure head is essentially a function of elevation above mean sea level (msl)
- Two ways to estimate mean atmospheric pressure head as a function of elevation:

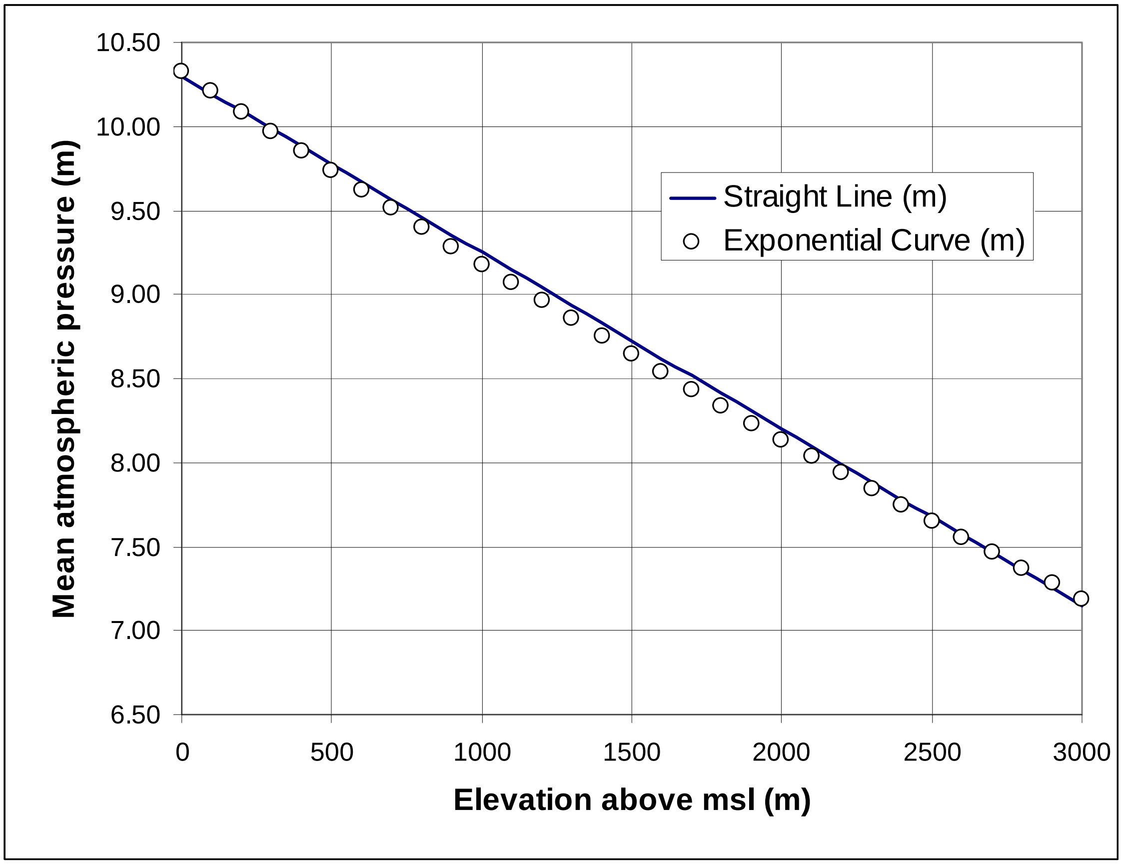
Straight line:

$$h_{atm} = 10.3 - 0.00105\,z \tag{247}$$

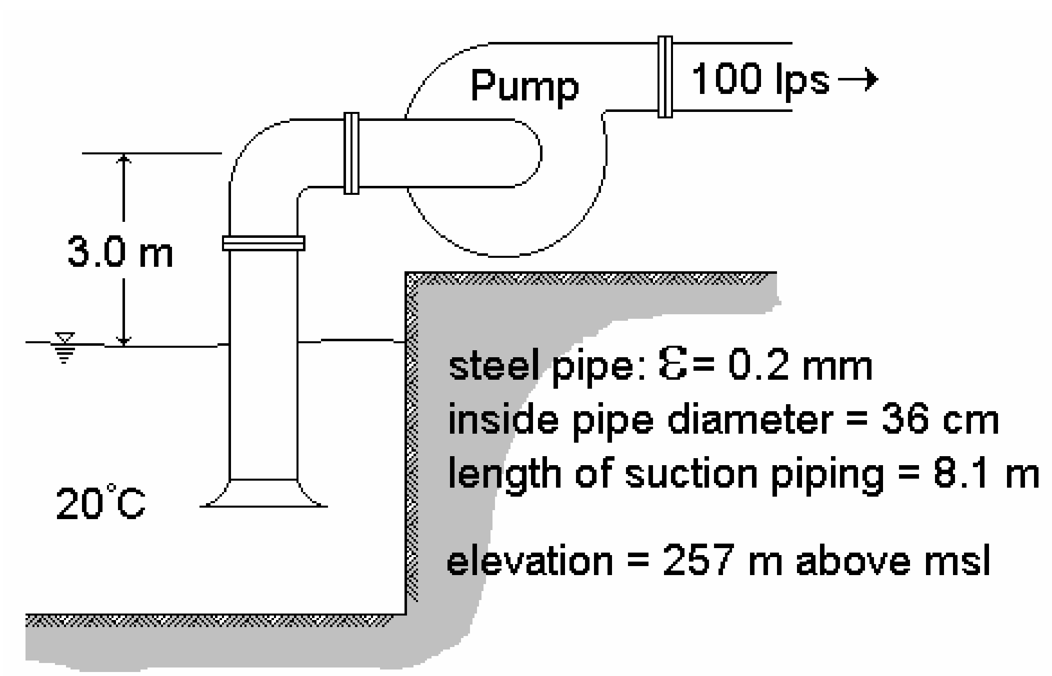
Exponential curve:

$$h_{atm} = 10.33\left(\frac{293 - 0.0065\,z}{293}\right)^{5.26} \tag{248}$$

where h_{atm} is atmospheric pressure head (m of water); and z is elevation above mean sea level (m)



V. Example Calculation of $NPSH_a$



1. Head Loss due to Friction

$$\frac{\varepsilon}{D} = \frac{0.2 \text{ mm}}{360 \text{ mm}} = 0.000556 \tag{249}$$

viscosity at 20°C, $\nu = 1.003(10)^{-6}$ m^2/s

flow velocity,

$$V = \frac{Q}{A} = \frac{0.100 \text{ m}^3/\text{s}}{\frac{\pi}{4}(0.36)^2} = 0.982 \text{ m/s} \tag{250}$$

Reynold's Number,

$$N_R = \frac{VD}{\nu} = \frac{(0.982)(0.36)}{1.003(10)^{-6}} = 353{,}000 \tag{251}$$

Darcy-Weisbach friction factor, f = 0.0184

velocity head,

$$\frac{V^2}{2g} = \frac{(0.982)^2}{2g} = 0.049 \text{ m} \tag{252}$$

head loss in suction pipe,

$$(h_f)_{pipe} = f\frac{L}{D}\frac{V^2}{2g} = 0.0184\left(\frac{8.1}{0.36}\right)(0.049) = 0.0203 \text{ m} \tag{253}$$

local losses, for the bell-shaped entrance, K_r = 0.04; for the 90-deg elbow, K_r = 0.14. Then,

$$(h_f)_{local} = (0.04+0.14)(0.049) = 0.0088 \text{ m} \tag{254}$$

finally,

$$(h_f)_{total} = (h_f)_{pipe} + (h_f)_{local} = 0.0203 + 0.0088 = 0.0291 \text{ m} \tag{255}$$

2. Vapor Pressure

for water at 20°C, h_{vapor} = 0.25 m

3. Atmospheric Pressure

at 257 m above msl, h_{atm} = 10.1 m

4. Static Suction Lift

- the center of the pump is 3.0 m above the water surface
- (the suction lift would be negative if the pump were below the water surface)

5. Available NPSH

$$NPSH_a = h_{atm} - h_{vapor} - \left(h_f\right)_{total} - h_{lift} - \frac{V^2}{2g} \tag{256}$$

$$NPSH_a = 10.1 - 0.25 - 0.0291 - 3.0 - 0.049 = 6.77 \text{ m}$$

VI. Relationship Between NPSHr and NPSHa

- If $NPSH_r < NPSH_a$, there should be no cavitation
- If $NPSH_r = NPSH_a$, cavitation is impending
- As the available NPSH drops below the required value, cavitation will become stronger, the pump efficiency will drop, and the flow rate will decrease
- At some point, the pump would “break suction” and the flow rate would go to zero (even with the pump still operating)